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The Brightness Scale of Exterior Scenes and the Computation of Correct Photographic Exposure*

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I. INTRODUCTION

IN any treatment of the photographic tone reproduction problem, where it is desired to determine the perfection with which the tonal characteristics of a given scene can be reproduced by the photographic process, it is necessary to know the brightness scale of the scene. If attention is centered solely on the objective phase of tone reproduction, it is not necessary to know the absolute brightness values but only the ratio of the maximum to the minimum brightness. This ratio is sometimes referred to as the "contrast" of the scene. A more definite and

* Communication No. 813 from the Kodak Research Laboratories.

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satisfactory term, which will be used consistently throughout this discussion, is the *brightness scale of the object*, BS_0. The word contrast is used qualitatively in several different senses in the literature of photography and has been defined quantitatively by one of us[1] for use in specifying a characteristic of photographic printing papers; hence, it seems desirable to avoid its use as a synonym for *brightness scale* in referring to a characteristic of an object or collection of objects of which a photographic reproduction is to be made.

If, in addition to the objective phase, the subjective phase of tone reproduction is being considered, a knowledge of the absolute brightness values is necessary. Furthermore, if it is desired to compute the exposure conditions (exposure time, lens aperture, etc.) which will result in the rendition of the object brightnesses at specified points on the density-log exposure characteristic of the negative material, it is imperative that the absolute values of these brightnesses be known.

A search of the photographic literature reveals but few data relative to the brightness and brightness distribution in exterior scenes. This is somewhat surprising in view of the vital importance of such information to the adequate treatment of tone reproduction and exposure problems. Hurter and Driffield[2] in connection with their classical work in the field of photographic sensitometry, concluded that the brightness scale of an average scene is approximately 30. So far as we are aware, no further data on this subject appear until the work of Mees[3] who concluded, from a series of photometric measurements made on a few exterior scenes, that the average landscape has a brightness scale of approximately 20. Scheffer,[4] who was interested in the brightness scale of scenes from the standpoint of photographic sensitometry, considered the item of greatest interest to be not the average but the maximum brightness scale. He stated that brightness scales as high as 100,000 may frequently be found in exterior scenes, but gave no estimate of the statistical average. Goldberg[5] stated in 1922 that a careful search of Eder's *Handbuch* yielded no important information on this subject other than that mentioned above. In his own monograph, "The structure of photographic images," he devotes considerable space to the discussion of the brightness and brightness differences existing in various types of photographic subjects. He states that a study of some three hundred scenes indicates that the average brightness scale is approximately 30.

Some years ago one of us made photometric measurements on the brightness distribution in a small group of exterior scenes. These were mostly of the landscape type, including some beach, open landscape, and woodland scenes. The group did not contain a sufficient number of examples so that the average could be considered as very representative of a wide variety of exterior scenes. However, they are indicative in a general way of the characteristics of such scenes. The lowest brightness scale, 10, was found in a beach scene on a cloudy day, while the greatest brightness scale, 250, was found in a woodland scene in which the foreground contained a heavily shaded area and the highest brightness was that of a sunlit cloud seen over a small body of water through an opening in the trees. From this rather cursory study it was concluded that the average brightness scale in exterior scenes probably lies in the neighborhood of 40 or 50.

Somewhat later, one of us published a communication[1] dealing with the evaluation of the "contrast" of photographic printing papers. As a test object for this work an effort was made to choose an exterior scene having a brightness scale approximating the average mentioned in the previous paragraph. Photometric measurements on the chosen scene gave a brightness scale value of 38. All of the brightness measurements on exterior scenes made by us prior to and including the work just mentioned were made with a commercially available portable photometer equipped with a telescope attach-

[1] L. A. Jones, "The contrast of photographic printing paper," J. Frank. Inst. **202**, 177 (1926); **203**, 111 (1927); **204**, 41 (1927).

[2] V. C. Driffield, "The Hurter and Driffield System: Being a brief account of their photo-chemical investigations and method of speed determination," Photo-Miniature **5**, 337 (1903); F. Hurter and V. C. Driffield, "Latitude in exposure and speed of plates," Photography, July 13, 1893.

[3] C. E. K. Mees, "The calculation of exposure," Wilsons Photographic Magazine **50**, 561 (1913).

[4] W. Scheffer, *Die Grundlagen der Photographie* (Union Deutsche Berlagsgesellschaft, Berlin, 1917), p. 69.

[5] E. Goldberg, *Der Aufbau des Photographischen Bildes* (W. Knapp, Halle, 1922).

ment so that relatively small areas in distant scenes could be measured. Subsequently, it was found that the measurements made with this instrument were subject to a systematic error due to the presence of a relatively small amount of stray or "flare" light in the instrument. While the absolute amount of flare was low, it was sufficient to produce serious errors in the measurement of the brightness of small low brightness areas surrounded by relatively large areas of high brightness. Hence, the brightness scale values thus far mentioned as emanating from these Laboratories are too low.

The difficulty of completely eliminating flare light from easily portable brightness photometers is rather great. It is almost impossible to make a precise correction for the presence of flare in such an optical system since the amount present depends not only on the optical characteristics of the instrument itself, but also upon the brightness and brightness distribution within the scene and its surroundings. It is therefore exceedingly important that in any work of this kind the brightness-measuring instrument should be as free from flare effects as is possible.

The need for more voluminous and accurate information on the brightness characteristics of objects has long been felt by all workers interested in the subject of tone reproduction. Consequently, about two years ago these Laboratories initiated a program of work designed to yield a large volume of data on brightness distribution, particularly relative to brightness scale, in exterior scenes.

II. Field Work and Laboratory Work

The accumulation of such data is a slow and laborious process since portable measuring equipment must be carried into the field and operated under conditions which frequently are not conducive to personal comfort and precise reading. Furthermore, under many conditions of outdoor lighting, brightness values change rather rapidly. If the data finally accumulated are to represent a fair approximation to the average exterior conditions, they should extend over a period of at least a year so that seasonal conditions of all kinds are taken into account. In addition, it is desirable to collect such data from localities having a wide geographical distribution. The results being reported at this time were obtained over a period of about 18 months. The major portion apply to locations in the neighborhood of Rochester, New York, although some field trips into the hills and mountains of northeastern New York state and in other directions give a somewhat increased geographical distribution. However, it must be remembered that all of the data reported here relate to the type of exterior scenes and lighting conditions existing within a radius of two or three hundred miles of Rochester, New York.

This collection of pictures contains a wide variety of scenes, including beach scenes, snow scenes, distant landscapes, street scenes, gardens, architectural subjects, exterior portraits, and portrait groups, etc.

An effort was made to obtain a fair distribution of subjects with respect to characteristics of lighting. As mentioned previously, however, the instability of exterior lighting conditions under partially cloudy and hazy weather conditions made it difficult to obtain reliable readings. A rather large majority of the scenes studied, therefore, were taken under direct sunlight illumination. This does not imply a cloudless sky, for under many circumstances it was possible to obtain sufficient stability of brightness to make the necessary photographic exposure and brightness measurements even though a considerable portion of the sky contained clouds. Back-lighted scenes and scenes with very strong cross-lighting effects were, in general, avoided.

Up to the present time about 130 scenes have been photographed, and the necessary brightness measurements made. In all cases values of maximum and minimum brightness in the area photographed were made and frequently several areas spaced along the entire tonal scale of the subject were measured. In addition, with but few exceptions, a reading of integrated brightness was made with a commercial exposure meter positioned at the camera with the plane of the light-sensitive surface being approximately perpendicular to the optical axis of the camera.

A. The brightness photometer

In choosing a portable photometer for this type of work, great care must be exercised. Of

prime importance, as previously mentioned, is the reduction of flare light within the optical system to the practical minimum. The instrument, of course, should be light and compact. The operations necessary in obtaining brightness readings should be simple and convenient for the operator in order to insure satisfactory and rapid work in the field. Since in many cases it is necessary to measure the brightness of relatively small areas at great distances from the point of observation, the photometer must be of the telescopic type in which a fairly large image of a distant object can be formed in the photometric field.

After considering the characteristics of the various instruments, we chose the one described by Luckiesh and Taylor.[6] With this instrument it is possible to measure a brightness varying from 0.0025 foot-lambert to 50,000 foot-lamberts. The comparison source is a small seasoned tungsten filament lamp operated from two small dry cells, the voltage applied to the filament terminals being controlled by a suitable rheostat and measured by means of a small built-in voltmeter. The objective lens is mounted at the inner end of a long narrow tube which serves to limit the angular field within which light can enter the lens. This serves as a very satisfactory lens hood and, when the interior of this lens mounting tube is carefully blackened by lining the tube with dead matte black paper, the amount of flare light associated with the image formed by the objective lens in the photometric field is very small. The photometric field is obtained by illuminating two trapezoidal-shaped areas with light from the comparison lamp. The bases of these trapezoids are disposed close to each other and the image of the object of which the brightness is being measured is formed in the narrow slit between the bases of these comparison fields. The focal length of the lens and the dimensions of the optical system are such that the image of an object one foot wide at a distance of approximately 500 feet completely fills the space between the two parts of the comparison field. An image of an object of the same size at a greater distance does not completely fill the space between the two parts, but it is still possible to make fairly precise brightness measurements under such conditions.

Before the instrument was used for field work and at various times during the course of the work, the reliability of its indications was checked carefully in the laboratory on a 3-meter bench photometer of standard design.

When in use this instrument was mounted on a rigid tripod equipped with a panoramic and tilting head. By careful design and adjustment these two motions were made sufficiently smooth and free from backlash so that the instrument could be aimed with ease and certainty at the area to be measured. It is estimated that the probable error of single readings under field conditions was about ± 5 or possibly ± 10 percent, depending to some extent upon the absolute value of the brightness being measured. By taking several readings and averaging, the probable error was greatly reduced. In all cases the photometer was set up as close as possible to the tripod carrying the camera so that the brightness measurements were made from a point near to that from which the photograph was taken.

B. The camera and lens

In order to facilitate the precise measurement of density in the negative corresponding to relatively small areas in the object, it is desirable that the negatives be relatively large. In choosing a camera and negative size, a compromise must be made between the desirable large size of negative and the size and weight of the equipment convenient for use in field work. It was decided to adopt 5 in.$\times$7 in. as the negative size in this investigation. A Speed Graphic camera equipped with a Bausch and Lomb VIIa Protar lens was chosen. This combination gives a system having relatively little flare light incident on the photographic material. In Fig. 1 is shown a vertical cross section through the optical axis of the camera. The drawing is made precisely to scale so that the relative dimensions of the various components can be appreciated.

The lens consists of 8 elements, 4 of which are cemented into each of two components as shown. The assembled lens, therefore, has four free glass-air surfaces. These two components are mounted in the usual barrel equipped with iris

[6] M. Luckiesh and A. H. Taylor, "A brightness meter," J. Opt. Soc. Am. **27**, 132 (1937).

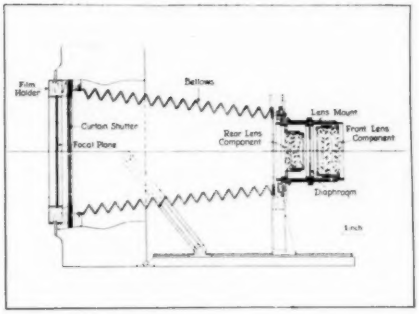

FIG. 1. Vertical section through the optical axis of the camera.

diaphragm. The focal length of the complete lens assembly is $10\frac{7}{8}$ inches operating at a maximum aperture of $f/7.7$. The 5 in.×7 in. image plane has a horizontal angular extension of 35 degrees and a vertical angular extension of 26 degrees, the angle subtended by the diagonal being 44 degrees. It will be noted that the bellows, when extended so that the lens is focused at infinity, has deep corrugations and is located at a relatively great distance from the optical axis. Such a structure tends to diminish the amount of flare light on the image plane due to interreflection within the body of the camera. This structure gives relatively low flare as compared with smaller, more compact designs which are used to so great an extent in practice, especially in the amateur field. The camera is equipped with the usual focal-plane shutter which is used in controlling exposure time.

C. Negative materials

All of the photographs of scenes in which brightness measurements have been made were obtained by using high speed panchromatic materials. Photographs of 115 of the scenes were made on Eastman Super Sensitive Panchromatic Film with the remaining 11 being made on Eastman Panatomic Film. While the spectral sensitivities of these two materials are not identical with that of the human eye, they are sufficiently near such sensitivity that the photographic brightness scale of the exterior scenes is, in general, quite close to the visual brightness scale.

The exposed materials were processed under precisely controlled conditions. A sensitometric developing machine[7] modified to carry the 5 in. ×7 in. sheets of negative material was used. A standard commercial developer formula, DK-50, was employed in processing these materials, and development was carried out to yield a normal product, gamma being approximately 0.8.

Along with each sheet of exposed negative material a sensitometric strip was developed. This was exposed in an intensity-scale sensitometer, the time of exposure being 1/25 second. This exposure time is approximately the same as the average exposure time used in making the photographs in the field. From density measurements made on these sensitometric strips, and by averaging all of the data for a given negative material, a precisely determined density-log exposure curve for each negative material was obtained, this characteristic, obviously, being representative of the processing conditions under which the negatives themselves were developed.

D. Density measurements

All density measurements on negatives and sensitometric strips were made in terms of diffuse density. For this purpose a standardized densitometer was employed, care being taken to be sure that all adjustments were such as to give readings of diffuse density corresponding closely with the primary standard values for this quantity. It has been shown by previous work[8] that these measurements of diffuse density approximate very closely the effective contact printing density of the silver deposits under consideration.

III. EXPERIMENTAL RESULTS

As a result of this field and laboratory work, there is now available for analysis a voluminous supply of experimental data. As a preliminary to a systematic study of these data, it may be well to consider briefly the character and the source of the various items of information.

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[7] L. A. Jones, M. E. Russell, H. R. Beacham, "A developing machine for sensitometric work," J. Soc. Mot. Pict. Eng. **28**, 73 (1937).

[8] A. M. Koerner and C. Tuttle, "Experimental determination of the photographic density," J. Opt. Soc. Am. **27**, 241 (1937).

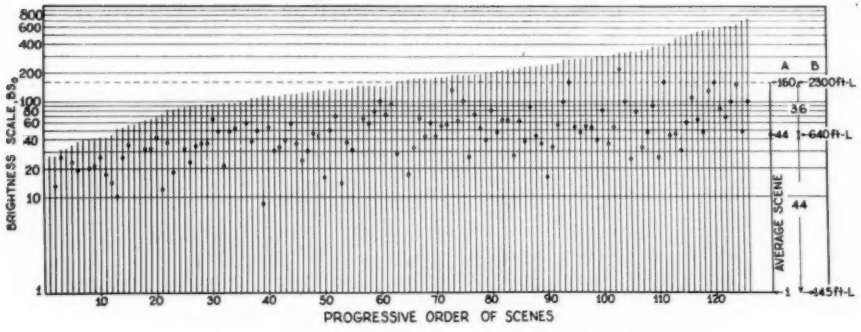

FIG. 2. Measured brightness scale values, outdoor scenes.

A. Measurements made on the objects

These include the values of the *minimum* and *maximum* object brightness as determined by the use of a visual brightness meter. The ratio of maximum to minimum object brightness gives the *brightness scale of the object*, BS_0. It should be pointed out that for many purposes the logarithms of these brightness values are more convenient and significant than the brightness values themselves. This is particularly true in obtaining the statistical average of the various groups of data. One reason for the use of the logarithmic method of expressing these values is that the human visual mechanism responds logarithmically to stimulation, that is, the sensation resulting from the stimulation of the eye by radiant energy is, at least over a considerable range of brightness values, directly proportional to the *logarithm of the stimulus* rather than to the *stimulus*.

One other measurement was made in the field on the scenes photographed by using a commercial exposure meter of the integrating type. This measurement was made from the camera position by holding the meter so that the surface of the sensitive receiver was approximately perpendicular to the optical axis of the camera. There are a number of such meters commercially available and because of variations of structural peculiarities among these, such as acceptance, angle, they do not in general give indications which bear a constant relation to specific brightness characteristics of a scene, such, for instance, as maximum, minimum, or average brightness. The value of integrated brightness obtained by the particular instrument used is designated as $B_0(M)$. The four values obtained by direct measurements on the object may be summarized as follows. The abbreviation or symbol at the left of each line will be used to designate the various quantities in further discussions and tabulations of data. All brightness values are expressed in foot-lamberts.

B_0min. = minimum object brightness,
B_0max. = maximum object brightness,
$B_0(M)$ = the integrated brightness of the scene,
BS_0 = brightness scale of the object,
BS_0 = B_0max./B_0min.

B. Graphic representation of the BS_0 data

Figure 2 shows all of the measured brightness-scale values. For convenience a semilogarithmic paper was used in plotting this graph. Each vertical line represents the magnitude of brightness scale for one scene. These lines are arranged without regard to classification under scene types and illumination levels in progressively increasing magnitude of brightness scale. The final group of negatives used for this work contains 126 members. It will be seen that the minimum brightness scale, at the extreme left of Fig. 2, is 27. This increases progressively to a maximum at the extreme right of 760. The average* brightness scale is 160, as shown in column *A* at the extreme right of Fig. 2. The ratio of the maximum to the average integrated brightness is 3.6, while the average integrated value is 44 times the average minimum object brightness. In column *B* are given these aver-

* All averages are made by using the log BS_0 values.

aged values in absolute terms, the minimum being 14.5 foot-lamberts, the maximum, 2300 foot-lamberts, and the integrated reading, 640 foot-lamberts.

The frequency of occurrence of the various values of brightness scale is shown in Fig. 3. As is customary in the construction of such curves, the abscissa, representing the brightness scale values, was divided into equal increments whose mid-points, in this case, progressed logarithmically in magnitude. The number of scenes falling within each increment or group has been plotted against the arithmetical values of the mid-points onto semilogarithmic paper. The curve as drawn is the Gaussian function which best fits the data. The average as shown in Fig. 2 is indicated by the arrow which falls at the peak of the curve. The failure of the data to fit this curve perfectly and the presence of what may appear to be secondary maxima may be attributed more to the lack of a sufficiently large number of scenes than to any fundamental dissymmetry of distribution in the frequency of occurrence of brightness-scale values in exterior scenes. It can be expected that, as the volume of brightness-scale data increases, the distribution data may conform more nearly with the Gaussian curve.

The average brightness-scale value is appreciably higher than that which has been previously reported as a probable average by these Laboratories and also by other workers in the field. A careful inspection of the photographs leads us to believe that they are fairly representative of average scenes and average illumination conditions within the vicinity of Rochester, New York. The higher brightness-scale average can be attributed largely to the use in this work of a brightness photometer which is almost completely free from the effects of flare light and, as stated before, it is now known that some of the previous work done in these Laboratories employed a photometer in which some flare light was present.

Figures 2 and 3 show completely all of the information obtained in this work bearing directly upon the question of brightness scale in exterior scenes. In later sections these same data will be discussed in connection with measurements of the magnitude of flare and its influence upon the image illumination scale under various conditions. It should be remembered that the negative material is only called upon to render the illumination scale in the camera image and is seldom, except in very unusual conditions, required to render the brightness scale of the object.

C. Measurements made on the negatives

For the purpose of this paper, interest centers in two or possibly three measurements made on each negative. The density of an area in the negative corresponding to the brightest object in the scene was measured and also the density in the negative corresponding to the object in the scene having the lowest brightness. In addition, a measurement of the fog density was made on each negative. The fog values vary so little that for all practical purposes an average value of 0.11 may be used. The difference between *maximum* and *minimum* negative density gives the *density scale of the negative*, DS_n. The density measurements were made in terms of diffuse density on the same instrument that was used for measuring the densities of the sensitometric strips. The three values applying to the negatives are as follows:

$D_n\text{min.}$ = minimum negative density,
$D_n\text{max.}$ = maximum negative density,
DS_n = density scale of the negative,
$DS_n = D_n\text{max.} - D_n\text{min.}$

D. Derived values

From the sensitometric data relative to the negative materials and the values obtained from measurements on the object and on the corre-

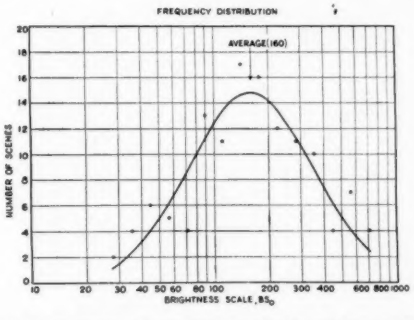

FIG. 3. Frequency of occurrence of the various values of brightness scale.

TABLE I. *Brightness data relative to the scenes of group 3. This group consists of near-by, sunlit, front-lighted, unshaded scenes.*

Scene No.	B_0min.	B_0max.	BS_0	$B_0(M)$	IS_i	FF	cfB_0min.	ΔB
7	12	2100	175	380	140	1.25	15	3.0
8	9.2	3600	390	240	290	1.35	12.5	3.3
18	6.0	2000	330	1300	62	5.30	32	26
27	18	2950	165	—	43	3.80	68	50
31	8.2	4000	490	500	205	2.40	19.5	11.3
32	8.2	1200	145	—	47	3.10	25.5	17.3
34	5.8	2250	390	940	58	6.70	39	33.2
39	14.3	2050	145	940	78	1.85	26.5	12.2
45	14.3	1800	125	350	91	1.35	19.5	5.2
60	13.3	3200	240	570	140	1.70	22.5	9.2
72	11.6	3500	300	940	135	2.20	25.5	13.9
73	22	7700	350	720	205	1.70	37	15
75	15.2	3200	210	720	91	2.30	35	19.8
76	12.5	1800	145	720	69	2.10	26	13.5
77	5.6	3200	570	720	210	2.70	15	9.4
78	16	3900	245	570	150	1.65	26.5	10.5
79	14.3	1900	135	720	51	2.65	38	23.7
80	31	1400	45	440	35	1.30	40	9.0
81	8.2	1100	135	570	29	4.70	39	30.8
83	22	1600	73	260	35	2.10	46	24
86	12.4	2000	160	350	74	2.15	26.5	14.1
88	17.3	1200	69	720	48	1.45	25	7.7
89	12.4	1600	130	570	47	2.75	34	21.6
90	14.9	620	42	260	33	1.25	18.5	3.6
98	27	2600	96	1300	66	1.45	39	12
116	19	2600	135	270	85	1.60	30	11
119	13	5400	420	570	225	1.85	24	11
134	15.3	2000	130	660	54	2.40	37	21.7
142	12.5	3100	250	410	115	2.20	27.5	15
151	11.5	1400	120	410	41	2.90	33	21.5
153	7.2	4500	620	500	200	3.10	22.5	15.3
154	12.7	3100	245	200	150	1.65	21	8.3
159	17	3400	200	660	56	3.60	61	44
163	13.5	8000	590	2100	100	5.90	80	66.5
165	10.6	2400	225	660	47	4.80	51	40.4
167	15.3	1700	110	750	49	2.25	34	18.7
174	6.7	2300	340	520	100	3.40	23	16.3
175	17	3000	175	1000	51	3.40	58	41
Min.	5.6	620	42	200	29	1.25	12.5	3.0
Max.	31	8000	620	2100	290	6.70	80	68
Av.	12.5	2400	190	580	80	2.40	30	15
Ratio	5.5	13	15	10.5	10	5.4	6.4	22.5

sponding negative, certain other valuable information can be deduced. The first derived value to be considered is that of *the illumination scale of the image*, IS_i. The mode of determining this value is illustrated by Fig. 4. Curve 1 is the sensitometrically determined characteristic curve for the negative material on which a particular negative was made. The maximum and minimum density values read from this negative are laid off on the density scale at the right of the figure and horizontal lines through the points thus established cut curve 1 at points A and B. Perpendiculars dropped from these points to the log E_x axis give the values of the maximum and minimum exposure to which this negative must have been subjected in the camera. Since the exposure time is constant at all points on the surface of the negative material, these exposure values are directly proportional to the corresponding image illuminations, I_i. The illumination scale of the image is defined as the ratio of the maximum to the minimum image illumination.

$$IS_i = I_i\text{max.}/I_i\text{min.}$$

The values of illumination scale determined in this manner for the various scenes are shown in the sixth column of Tables I and II.

A comparison of the values of IS_i with those of BS_0 shows that the former is always less than the latter. In some cases the difference is relatively small, while in other cases the difference is very appreciable. The shortening of the image illumination scale as compared with that of the brightness scale of the object is due to the presence of nonimage forming light in the camera, this light being usually referred to as "flare light" or "flare." It arises from several sources such as the interreflection between the glass-air surfaces of the lens system, reflection from the interior of the lens mount, shutter, and diaphragm blades, and the interior surfaces of the camera, etc. If no flare light is present the value of IS_i is equal to the value of BS_0.*

The relation between the brightness scale of the object and the illumination scale of the image serves as a convenient and useful method of specifying the magnitude of flare present under any particular set of conditions, and for the purposes of this discussion the *flare factor* is defined as follows:

$$FF\ (\text{flare factor}) = BS_0/IS_i.$$

Values of the flare factor for the various scenes are given in the seventh column of Tables I and II. An inspection of Table I will show that this flare factor varies over a considerable range. The same lens-camera combination was used in

* It should be pointed out that this statement depends upon the validity of the assumption that the brightness scale of the object as evaluated visually is equal to that evaluated photographically in terms of the particular negative material being used. When the spectral sensitivity of the negative material is considered and, further, when we remember that, in general, the quality of the light reflected from the brightest and darkest objects does not depart very markedly from the spectral distribution of the light used in determining the sensitometric characteristic for this material, we feel that the assumption is justified. Whether or not this assumption is correct, curve 2 does give the true relation between negative density and object brightness.

making all of these pictures. It is obvious, therefore, that the magnitude of the flare factor is not determined entirely by the optical and mechanical characteristics of the camera assembly, but depends also upon the nature of the scene being photographed and its environment. The aspect of scene and environment which determines flare factor is of course the distribution of brightness in the scene and in the environment of the scene. While it is true that for a given distribution of brightness in scene and environment the actual amount of flare light incident on the focal plane of the camera is dependent upon the average brightness level in the scene and its environment, the flare factor as defined is not dependent upon this average level. While it might be possible by means of a complete analysis of the brightness distribution in the scene and its environment to compute the flare factor, there seems to be no possibility of making a single simple measurement from which the desired value could be deduced.

It must be remembered that the values of image illumination as determined by the graphic method illustrated in Fig. 4 are of the *total* illumination characteristics of the image which, at any point on the focal plane, is made up of two components one due to image forming light from the corresponding point in the object and the other to nonimage forming flare light arising from various sources. It is frequently necessary to separate these two components and for this purpose the following nomenclature will be used:

$I_i = I_{i0} + I_{if}$,
I_i = total image illumination,
I_{i0} = image illumination due to object alone,
I_{if} = image illumination due to flare alone.

It is sometimes convenient to regard the presence of flare light in the camera as equivalent, in its effect upon the realizable object brightness scale, to the addition of a constant increment of brightness, ΔB, to all the actual existent brightnesses in the scene. If this hypothetical scene, the brightnesses of which are obtained by adding a constant ΔB to all the brightness values found in a real scene, is photographed with a camera-lens system having zero flare, the distribution of brightness in the image formed on the focal plane of the camera will be the same as if the real scene were photographed with a camera-lens system having a flare factor as measured under the conditions of the actual scene and its environment such as to result in the value of ΔB used in the construction of the hypothetical scene.

From the data now available it is possible to compute for each of the scenes studied the value of ΔB. This is given by the following equation:

$$\Delta B = \frac{(I_i\text{min.} \cdot B_0\text{max.}) - (I_i\text{max.} \cdot B_0\text{min.})}{I_i\text{max.} - I_i\text{min.}}.$$

Computed values of ΔB for each of the scenes are shown in the last column of Tables I and II. It will be noted that these values in general are very small, in fact almost negligible as compared to the values of B_0max., but in many cases are large as compared to the values of B_0min. If it is desired to determine for any given scene the values of effective minimum and maximum object brightness, it is only necessary to add the value of ΔB to each of the two values. Thus

$$B_0\text{max.} + \Delta B = ef\,B_0\text{max.},$$
$$B_0\text{min.} + \Delta B = ef\,B_0\text{min.}$$

The values of the effective minimum brightness

TABLE II. *Summary of the average brightness values applying to all groups of scenes.*

	B_0MIN.	B_0MAX.	BS_{B_0}	$B_0(M)$	IS_i	FF	$ef B_0$MIN.	ΔB
Summary of group 1 scenes								
Min.	20	1500	27	720	18	1.15	54	8.0
Max.	160	11500	285	5700	91	3.30	255	139
Av.	55	4500	81	1650	43	1.90	105	43
Ratio	8.0	7.7	10.5	7.9	5.1	2.85	4.7	17.5
Summary of group 2 scenes								
Min.	8.4	820	35	270	22	1.30	20.5	9.2
Max.	76	9200	340	3000	155	4.80	165	109
Av.	24.5	3150	130	910	56	2.30	56	29.5
Ratio	9.0	11	9.7	11	7.0	3.7	8.1	12
Summary of group 3 scenes								
Min.	5.6	620	42	200	29	1.25	12.5	3.0
Max.	31	8000	620	2100	290	6.70	80	68
Av.	12.5	2400	190	580	80	2.40	30	15
Ratio	5.5	13	15	10.5	10	5.4	6.4	22.5
Summary of group 4 scenes								
Min.	2.0	710	115	53	32	1.25	5.6	1.1
Max.	11.6	4000	750	720	440	8.70	39	34.5
Av.	4.6	1600	345	250	135	2.55	11.5	5.9
Ratio	5.8	5.6	6.5	13.5	13.5	7.0	7.0	31.5
Summary of group 5 scenes								
Min.	0.82	55	27	38	22	1.15	1.1	0.22
Max.	50	3700	640	2300	350	9.50	79	65
Av.	6.3	1050	165	320	60	2.75	17.5	8.9
Ratio	61	67	23.5	61	16	8.3	72	295
Summary of all scenes								
Min.	0.82	55	27	38	18	1.15	1.1	0.22
Max.	160	11500	750	5700	440	9.50	255	139
Av.	14.5	2300	160	640	68	2.35	34	17
Ratio	195	210	28	150	24.5	8.3	230	630

for each of the scenes are tabulated in the next to the last columns of Tables I and II.

It must be remembered that curve 1, which is determined sensitometrically, shows the relation between density and the exposure incident upon the negative material which of course in the camera is determined by the total image illumination, I_i. In the presence of flare this curve cannot give the relationship between negative density, D_n, and object brightness, B_0. For the purposes of many problems in tone reproduction it is necessary to know the latter relationship and we now have available information for its determination.

At the top of Fig. 4 is established the scale of log B_0 which, for the time being, we may regard as showing only relative values. The point A' is now established on the horizontal line through $D_n$max. to the left at point A, the separation being given by

$$\log \frac{B_0\text{max.}+\Delta B}{B_0\text{max.}}.$$

The point D is located on the horizontal line through $D_n$min. to the left of point B, the separation being given by

$$\log \frac{B_0\text{min.}+\Delta B}{B_0\text{min.}}.$$

These points represent the extremities of the desired relationship between negative density and log B_0. Additional points on this curve may be located by taking other values of negative density and determining in a manner similar to that used in locating A' and D the horizontal displacement from curve 1, using of course the same value of ΔB in all cases.

We may now convert the relative log B_0 scale into one showing actual brightness units. It is apparent that the B_0 value for the point A' must be the measured maximum brightness of the scene being considered, namely, B_0max. Furthermore the separation between the points D and A' must be equivalent to the log of the brightness scale, log BS_0, and hence the point D must correspond to the minimum brightness of the scene, B_0min. Hence if the log B_0 scale be so adjusted that the value read from this scale

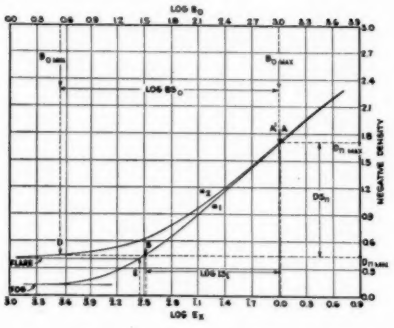

FIG. 4. Curve 1, density-log exposure characteristic of the *negative material;* curve 2, between points D and A, is the density-log B_0 characteristic of a particular *negative.*

corresponding to the point A' is the logarithm of B_0max. as measured, then for all points of the object the actual value of brightness may be read. In the particular case shown, log B_0 for A' is 3.0 and hence the maximum brightness of this particular scene must have been 1000 foot-lamberts.

Furthermore, it is apparent from the construction that the flare factor may now be expressed in terms of the separations between the points D and B and points A' and A. Thus

$$\log FF=\log \frac{B_0\text{min.}+\Delta B}{B_0\text{min.}}-\log \frac{B_0\text{max.}+\Delta B}{B_0\text{max.}}.$$

As shown in Fig. 4 the separation between the points A' and A is quite small but, in fact, this represents the maximum separation of any of the actual cases which have been studied in this work, this maximum separation actually being somewhat less than 0.02. For all practical purposes, therefore, it is quite permissible to regard points A' and A as coincident, in which case log FF is given directly by the separation between the points D and B. In fact, this approximate graphic construction does not introduce errors in excess of approximately 2.5 percent in any of the cases we have thus far studied, in many of which the amount of flare light present is relatively high. It must be recognized of course that theoretically the flare factor as defined by us may vary from unity (1.0) to infinity in case of a scene in which the minimum brightness is zero. As a matter of fact, we have

never studied a scene in which the minimum brightness is not finite and measurable. The theoretical case of infinite flare factor therefore seems to be of more academic than practical interest.

It is necessary to distinguish very carefully between the two curves shown in Fig. 4. Curve 1 should be referred to in all cases as the characteristic ($D_x - \log E_x$) curve of the *negative material*, while curve 2 should be referred to as the characteristic ($D_n - \log B_0$) curve of the *negative*. If for any purpose it is desired to determine the magnitude of a negative density difference, ΔD_n, corresponding to some object brightness difference, $\Delta \log B_0$, curve 2 must be used. Thus, in dealing with tone reproduction problems which involve a consideration of gradients, curve 2 gives the desired information.

In the application of these data to the evaluation of tone reproduction quality and to the determination of effective camera speeds of negative materials by means of a gradient criterion, a knowledge of the gradient characteristics of curves 1, $dD/d \log E_x$, and 2, $dD/d \log B_0$, is necessary. Of immediate interest are the gradient values at the limits of the used portions of these curves. Such gradient values will be referred to as limiting gradients and to distinguish between the gradients at the two ends of a given curve use will be made of the two words "shadow" (s) and "highlight" (h), as identifying the regions of the object tonal scale to which the limiting gradient applies.

In previous work the symbol θ has been used as an abbreviation of the phrase "limiting gradient." This usage will be continued. We shall also continue the convention of using x as a subscript in referring to a characteristic of the *negative material* and of y as a subscript when the *positive material* is referred to. Likewise, the subscripts n and p will be used to indicate characteristics of the *negative* and the *positive*, respectively. The abbreviation or symbol $h\theta_n$ is read, therefore, the limiting gradient of the *negative* in the highlight region; while the abbreviation $h\theta_x$ means the limiting gradient of the *negative material* characteristic in the highlight region. Thus, terms carrying the subscript x refer to characteristics of curve 1, while those carrying the subscript n refer to those of curve 2.

It is frequently desirable to determine values of the average gradient of certain portions of the characteristic curve. The Greek letter ϕ, as previously, has been used as a symbol for this quantity. Here, again, the subscripts x and n distinguish between characteristics of the negative material and those of the negative. Thus, for the situation illustrated in Fig. 4:

$$\phi_x = \frac{DS_n}{\log BS_0 - \log FF} = \frac{DS_n}{\log IS_i},$$

$$\phi_n = DS_n / \log BS_0.$$

Because of flare light in the camera and lens system as used in the particular environment existing when the negative was made, the gradient at B is *not* the limiting gradient of the *negative* in the shadow region. This limiting gradient is given by the slope of curve 2 at the point D. It can be shown by simple geometrical treatment that the gradient at point D is equal to that at point B divided by the flare factor. Thus,

$$G_D = G_B / FF.$$

Curves 1 and 2 are practically coincident at the point A in the highlight region. While the gradients of the two curves are slightly different at this point, this difference is so small as to be negligible for all practical purposes and hence it is unnecessary, in general, to distinguish between the highlight limiting gradients of the *negative material* and the *negative*. Curve 2, if extended to the left, becomes asymptotic to the horizontal line marked "flare" which indicates the magnitude of density, E, which will be found on this negative corresponding to an object area of zero brightness. This of course can be evaluated in terms of brightness units if necessary, but such evaluation is of little importance. The most significant and useful method of expressing the magnitude of flare in practice is in terms of either the flare factor or the logarithm of the flare factor.

If $\log FF$ is subtracted from $\log BS_0$, the remainder is the logarithm of the illumination scale, $\log IS_i$, of the image, formed on the focal plane.

The various values derived by this graphic computation may be summarized as follows:

IS_i = illumination scale of the camera image,
FF = flare factor,
$ef\ B_0$ min. = the effective minimum object brightness
$= B_0$ min. $+ \Delta B$,
θ = limiting gradient,
$h\theta_n$ = limiting gradient of the negative in the highlight region,
$s\theta_x$ = limiting gradient of the negative material in the shadow region,
$s\theta_n$ = limiting gradient of the negative in the shadow region,
ϕ = average gradient,
ϕ_x = average gradient of the negative material,
ϕ_n = average gradient of the negative.

IV. Classification of Scenes into Homogeneous Groups

An inspection of the data shows that the brightness values vary over a relatively great range from scene to scene. This is to be expected since the group includes pictures taken under a wide variation in illumination conditions and of very diverse scene types. For any individual scene it is of course possible to compute the exposure required to render any chosen brightness characteristic, such, for instance, as the minimum brightness, at a specified point on the characteristic curve of the negative material. However, it is obvious that many scenes resemble each other quite closely with respect to brightness levels, and hence if these scenes can be classified by inspection into relatively homogeneous groups, then it may be possible to recommend a single exposure value which will render satisfactorily all members of such a group. An attempt will therefore be made to set up a rational classification system which will enable us to assign each scene to one of a few homogeneous groups defined in terms of brightness characteristics and scene type.

From a consideration of the way in which exterior scenes differ among themselves in composition and lighting, such a classification according to three variables has been made: General illumination characteristics, direction from which the object or scene is photographed, spatial distribution of the scene elements (local illumination conditions).

A. General illumination characteristics

It seems reasonable to subdivide this factor into four conditions based upon the general level of illumination and upon the relation between the direct and diffuse components of the illumination incident upon the scene. These four subdivisions may be designated as follows: (1) sunlit, (2) haze, (3) light clouds, (4) heavy clouds.

The *sunlit* condition implies that the scene as a whole is illuminated by the unobscured sun and by diffuse light from the major portion of the sky hemisphere which may be cloudless or may contain some cloud formations. Illumination levels are high and shadows are strong and sharp.

The *haze* condition is encountered when the greater part of the sky is overcast with thin layers of condensed water vapor, or by dust in the atmosphere, or both. The ratio of the directional light from the sun to the diffuse light from the sky is lower than under the condition designated as sunlit. Shadows are less sharp, show softer edges, and have relatively more illumination thrown into the shadow areas by diffuse light from the sky areas. The general illumination level is about one-half of that encountered under the sunlit condition. Under these conditions the sun's disk is clearly visible.

The condition designated as *light clouds* is found when the major portion of the sky area is covered with a thin cloud layer sufficiently dense to make the sun's disk invisible. But frequently there is a relatively large bright spot surrounding the position of the sun, which acts as a source of somewhat unidirectional illumination. Some shadows are formed but they are very soft with poorly defined edges. The general illumination level is usually about one-half of that encountered under the condition designated as haze. The ratio of directional light to diffuse light is still lower than that found under hazy conditions.

The term *heavy clouds* is used to describe the condition when the sky is filled with dense dark cloud masses. The illumination incident on the scene is now almost completely diffuse. Shadows have largely disappeared. The directional component has practically vanished and the general level is about 10 percent of that found under sunlit conditions.

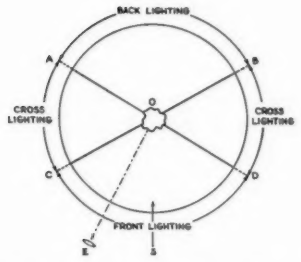

FIG. 5. Diagram illustrating the directional viewing and illumination conditions.

B. Directional viewing aspect

Under any circumstances where a part of the illumination incident upon the scene is unidirectional, that is coming from the unobscured or partly obscured sun, it is obvious that the brightness of the side facing the source will be greater than that of parts of the object which are partially or completely obscured from this direct illumination. Therefore, in predicting the necessary exposure in photography of such scenes, some consideration must be given to the direction of view relative to that from which the unidirectional illumination is incident on the elements of the scene. Figure 5 illustrates one possible method by which this direction effect can be divided into three significant aspects. In this figure, which is a projection on a horizontal plane of object, light source, and camera position, S indicates the source. The unidirectional illumination from this source is therefore incident on the object along the line SO.

Now if the camera, E, be located at any angular position between the points designated as C and D, the objects will be seen from the same general direction as that from which the unidirectional light is incident, and this condition is defined qualitatively as *front lighting*. There is considerable variation within this angular range so that the shadows cast by irregularities in the object may vary from relatively short, if the camera happens to be located near the line SO, to fairly long ones if it is located at either of the angular extremes.

If the camera is located between A and C or between B and D, the condition is referred to as *cross lighting*, the surface facing the camera being illuminated by light incident at high angles, resulting in long, distinct shadows.

If the camera is located between the points A and B, the condition is referred to as *back lighting*.

The three subdivisions under this variable therefore are as follows: (1) front lighting, (2) cross lighting, (3) back lighting.

C. Spatial distribution of objects

It is evident that objects may be distributed within the three-dimensional space in three directions, namely, horizontal, vertical, and longitudinal. The latter word is used to designate distribution along or parallel to the axis of the optical system. There is not much to be gained in classifying scenes according to the distribution of its elements in the vertical and horizontal directions. Distribution with respect to distance from the camera, however, is of considerable importance. Four subdivisions according to such distribution seem to be adequate, namely: (1) distant, (2) remote, (3) near-by, (4) close-up.

Compositions designated by the term *distant* contain few, if any, objects of interest near the camera. The scene is composed usually of a multitudinous collection of elements at relatively great distances and this entire collection of objects merges into the composition of the scene as a whole. Although certain elements of the composition are distinctly separable, attention is not centered on any particular one of the objects but is concerned largely with aspects such as balance of light and shade, rendition of distant mountains, rivers, lakes, etc., and possibly clouds in the sky. A scene falling in this category is illustrated in Fig. 6.

In scenes classified as *remote** the component parts are considerably closer to the camera than in distant scenes. Such scenes usually include at least a portion of sky. They may include groups of people but these are not sufficiently close to the camera to become objects of chief interest and hence are merely incidental to the composition as a whole. The scene in Fig. 7 is typical of this class.

The term *near-by* is used to designate scenes

* The general connotation of the word "remote" is almost synonomous with that of the word "distant." In this paper we arbitrarily define the word remote as applying to a grouping of objects intermediate between those which are close to the camera, near-by, and at a relatively great distance, extending in most cases from several hundred feet from the camera to the horizon.

FIG. 6. Typical distant, sunlit, front-lighted scene.

which are composed of objects which are relatively near to the camera. Figure 8 shows a typical example of this type. Usually such a composition contains a single object or group of objects on which attention is definitely fixed. While there may be other elements in the composition they are regarded largely as environmental or background elements. They frequently include small areas of sky but are characterized as a rule by the fact that the objects on which interest is centered occupy the major portion of the picture area. In this category will be found a very large proportion of all the pictures taken by the average amateur. They may include full-length portraits of an individual or of groups of individuals, pictures of single buildings, parts of a garden, etc.

The fourth subdivision is designated as *close-up*, of which a typical representative is shown in Fig. 9. Here there is a single object of maximum importance and interest. Everything else in the composition is subordinate and, in fact, this object is frequently so close to the camera that there is very little else visible in the reproduction. It is very easy to identify compositions falling within this category, e.g., the head and shoulders of a single individual or of a small group of individuals, a picture of a single rose or a vase of flowers which fills the greater part of the picture area.

Now, an inspection of a large number of scenes of the *distant* category falling within any one of the four general illumination groups, for instance, sunlit, indicates that a relatively high degree of homogeneity prevails with respect to brightness characteristics. This condition exists also for scenes in the *remote* category. When those classifiable as near-by or close-up are considered, however, it is found that a considerable heterogeneity of lighting characteristics exists within any one of the groups thus formed. A further analysis of this situation leads to the conclusion that the reason for this heterogeneity is that in scenes of these two types the *local* illumination conditions are profoundly influenced by the spatial distributions of the objects concerned. Thus, one element of the composition may be so located relative to others that it is shielded more or less completely from direct illumination by the sun or by large areas of sky. To deal with this condition we establish three subgroups, as follows: (a) unshaded, (b) light shade, (c) heavy shade. These terms indicate the *local* illumination conditions prevailing on the objects of chief interest in each case. Thus the general illumination on the scene as a whole may be describable as sunlit, and the

FIG. 7. Typical remote, sunlit, front-lighted scene.

objects of chief interest may be illuminated by sunlight plus a large portion of the sky. This local illumination condition is referred to by the term *unshaded*. On the other hand, the objects of chief interest may be obscured from direct sunlight but illuminated by relatively large areas of the sky. This is the condition referred to as *light shade* (the term *open shade* is sometimes used) and is illustrated by Fig. 10. The third condition, *heavy shade*, exists when the objects of chief interest are shielded from direct sunlight and also from the major portion of the sky, being illuminated by relatively small sky areas and by light reflected from adjacent objects. In the case of the general illumination conditions designated

as light clouds and heavy clouds, the local illumination conditions, unshaded, light shade, and heavy shade obviously imply that the objects of chief interest are illuminated by either very large, restricted, or very small sky areas.

This system of classifying scene characteristics is shown in the following outline:

A. General illumination conditions, *I*

	Unidirectional	Diffuse	Relative Illumination Level
1. Sunlit	80%	20%	100
2. Haze	60%	40%	50
3. Light clouds	30%	70%	25
4. Heavy clouds	0%	100%	10

B. Directional viewing aspect, *D*
   1. Front lighting
   2. Cross lighting
   3. Back lighting

C. Spatial distribution of objects, *S*
   1. Distant
   2. Remote
   3. Near-by
      a. Unshaded
      b. Light shade
      c. Heavy shade
   4. Close-up
      a. Unshaded
      b. Light shade
      c. Heavy shade.

All the scenes on which brightness measurements have been made were now grouped according to this classification. The inspection and classification was done independently by three observers. It was found that all scenes can be classed as front-lighted. Out of the total number of 126, 104 fell within the *sunlit* group of general illumination, the remaining 22 being scattered between the *haze*, *light cloud*, and *heavy cloud* categories. In some cases an observer encountered some difficulty in deciding into which of two adjacent groups a given composition should be placed. In these instances it was placed in both of the two adjacent groups so that it contributed to the final average values in both groups. With respect to spatial distribution of scene elements, 28 (of the sunlit group) were placed in the *distant* category, 39 in the *remote* classification, 38 in the *near-by unshaded group*, and 19 in the *near-by shaded* group.

FIG. 8. Typical nearby, sunlit, front-lighted, unshaded scene.

FIG. 9. Typical close-up, sunlit, front-lighted, unshaded scene.

V. Classified Numerical Data

It seems inadvisable to give a complete detailed tabulation of all the data bearing on the measured scenes and the resultant negatives. We have chosen, therefore, one of the four groups and in Table I are given all of the brightness measurements on the members of that group. The column headings are denoted by the symbols which have already been defined. At the bottom of the table are the minimum, maximum, and average values for each column. This group contains 38 scenes which may be described as *front-lighted, sunlit, near-by* compositions, in which the object or objects of chief interest are unshaded. It will be noted that the lowest value of B_0 min. is 5.6, while the highest is 31, the average being 12.5 and the ratio of maximum to minimum 5.5. From the standpoint of minimum object brightness this group therefore exhibits a rather high degree of homogeneity. When the value of maximum brightness is considered, we find that the spread

is somewhat greater and this applies also to the integrated brightness value in the column $B_0(M)$. In the last four columns of the table are the values derived in accordance with the previous discussion of Fig. 4. The average value of brightness scale, BS_0, for this group of objects is 190, while the average value for the image illumination scale, IS_i, is only 80, this reduction being due to the presence of flare light in the camera system. The average flare factor, FF, for this group, it will be noted, is 2.4, which, as pointed out previously is equal to the ratio of the average brightness scale to the average image-illumination scale.

In Table II are summarized the average values for two other homogeneous groups, group 1 containing front-lighted, distant, sunlit scenes and group 2, front-lighted, remote, sunlit scenes. Group 4 contains 19 scenes classifiable as front-lighted, near-by, sunlit scenes in which objects of principal interest are shaded either lightly or heavily. This group is later split into two subgroups depending upon the degree of shading. Group 5 contains all the scenes not classifiable in the previous four groups and represents a heterogeneous collection of scenes under haze, light cloud, and heavy cloud conditions. They differ among themselves in distant, remote, and near-by spatial distribution. At the bottom of the table is a summary for the 126 scenes used in this work.

It is interesting to note that the average B_0min. value decreases progressively and in an orderly manner from groups 1 to 4, inclusive. The average brightness scale increases progressively in these groups.

The flare factor increases systematically from group 1 through 2 and 3 to 4. The value of ΔB, however, decreases progressively from group to group. These two facts taken together mean that while, in general, a distant open landscape results in a higher absolute value of flare light on the focal plane, its effect in reducing the illumination scale of the image is less than in the case of a group 4 scene. This follows from the fact that the shadow brightness of a group 4 scene is relatively low and hence a small amount of flare light results in a greater proportionate reduction of the illumination scale of the image corresponding to an increase in the flare factor.

FIG. 10. Typical near-by, sunlit, front-lighted, lightly shaded scene.

VI. COMPUTATION OF EXPOSURE

A. Relation between object brightness and image illumination

It will be of interest now to consider the establishment of certain exposure conditions which will give satisfactory results when photographing scenes of known brightness characteristics as illustrated in Tables I and II. The first step in this operation is the development of a formula which will give the relation between the brightness of an object and the illumination in the image of that object formed on the focal plane by the lens system. This relationship has been studied by many workers and we shall mention only a few of the possible references at this time. Nutting[9] developed a very complete and rigorous formula for this purpose. A little later Moffitt[10] published a similar formula and Hardy and Perrin[11] in their exhaustive treatise, *The Principles of Optics*, also discussed the subject showing the relation in convenient form. Goodwin,[12] by making certain assumptions as to the probable statistical average values which are operative under a wide variety of practical conditions, reduced the expression to a somewhat simpler form which is useful for the computation of ex-

[9] P. G. Nutting, "On brightness and contrast in optical images," Astrophys. J. **40**, 33 (1914).

[10] G. W. Moffitt, "A method for determining the photographic absorption of lenses," J. Opt. Soc. Am. **4**, 83 (1920).

[11] A. C. Hardy and F. H. Perrin, *The Principles of Optics* (McGraw-Hill, New York, 1932), p. 409; A. C. Hardy, "The distribution of light in optical systems," J. Frank. Inst. **208**, 773 (1929).

[12] W. N. Goodwin, Jr., "The photronic photographic exposure meter," J. Soc. Mot. Pict. Eng. **20**, 95 (1933).

posure in photographic work. The complete formula for this relationship is as follows:*

$$I_{i0}=B_0\cdot\frac{F^2}{4v^2f^2}\cdot\cos^4\theta\cdot H\cdot T_g, \qquad (1)$$

in which the symbols have the following significance:

I_{i0} = image illumination (foot-candles),
B_0 = object brightness (foot-lamberts),
F = focal length,
v = image distance,
d = diameter of stop,
f = F/d, aperture ratio,
θ = angle of image point off axis,
T_g = transmission as limited by losses due to reflection and absorption by glass,
H = transmission at off-axis points as limited by barrel vignetting.

In case the object lies on the axis of the lens, the $\cos^4\theta$ and the H terms in this equation may be ignored, and if the object is at infinity the equation simplifies to:

$$I_{i0}=B_0\cdot T_g/4f^2. \qquad (2)$$

Objects on the average, however, are not at infinity and hence it seems desirable that a correction be made for the change in the relationship between I_{i0} and B_0 for finite object distances. It seems reasonable to assume that in amateur practice the average distance, u, of the object from the lens is of the order of 40 times the focal length of the lens. Since it is desired to obtain satisfactory exposure for objects lying off the axis as well as on it, account should be taken in a practical formula for this purpose of the angle subtended by such off-axis objects. Careful consideration of the angular dimensions subtended by average negatives leads to the conclusion that the average displacement from the optical axis of all object elements is probably about 15 degrees.

$$\cos^4 15^\circ=0.87.$$

In addition to this loss of brightness at off-axis points, there is an additional loss due to the vignetting by the barrel or tube in which the lens elements are mounted. If such a mounting is viewed from a point on the axis, it appears to be circular, but if the point of view is moved away from the axis, this opening becomes oval in shape, having a smaller area than the full circle. Assuming again that a point 15 degrees off the axis represents a satisfactory practical average, the loss due to this vignetting for an average lens is about 20 percent.

Some light from the object is lost because of reflection at the glass-air surfaces of photographic objectives and because of a little absorption within the glass itself. It is estimated that, on the average, modern lenses which frequently have six or even eight glass-air surfaces do not transmit as image-forming light more than 70 percent of the incident light. These various factors which require consideration in obtaining a probable average suitable for practice are:

(a) $\cos^4\theta=0.87$, $\theta=15^\circ$,
(b) $T_g=0.70$,
(c) $H=0.80$, $\theta=15^\circ$,
(d) Correction for finite object distance. For object distance, $u=40F$

$$\frac{F^2}{v^2}=\left(\frac{u-F}{u}\right)^2=0.95.$$

Inserting the values of these four factors in Eq. (1), the result becomes

$$I_{i0}=B_0\cdot 0.463/4f^2. \qquad (3)$$

If it be desired to express I_{i0} in meter-candles instead of foot-candles, then the equation becomes

$$I_{i0}(\text{m.c.})=B_0(\text{ft-L})\cdot 0.463\cdot 10.76/4f^2, \qquad (4)$$

which reduces to

$$I_{i0}=B_0\cdot 4.98/4f^2, \qquad (5)$$

$$I_{i0}(\text{m.c.})=5B_0(\text{ft-L})/4f^2. \qquad (6)$$

B. Exposure formula

The formula for expressing photographic exposure is

$$E=I\cdot t, \qquad (7)$$

where I (illumination) is in meter-candles and t is in seconds.

* This formula neglects one minor factor included in Nutting's treatment, this being the slight decrement of image brightness due to the fact that under some unusual conditions the illumination arising from an object point and incident upon the first surface of the photographic lens is not perfectly uniform.

C. Speed-number formula

In recent papers published by one of us in collaboration with C. N. Nelson[13,14] of these Laboratories, it has been shown that effective camera speeds can be determined satisfactorily by sensitometric methods using a gradient criterion. The formula for speed number becomes

$$S=1/E_{0.3\bar{G}}, \tag{8}$$

where S is speed and $E_{0.3\bar{G}}$ is the exposure (in meter-candle-seconds) at a point on the characteristic curve of the negative material where the gradient is 0.3 times the average slope of the characteristic curve over a 1.5 log exposure range.

D. Exposure-time formula

Combining Eqs. (6), (7), and (8) and solving for exposure time, t, the following is obtained:

$$t\ (\text{sec.})=4f^2/(5B_0\cdot S). \tag{9}$$

Since confusion in the interpretation of this formula frequently arises because of the neglect to specify definitely the photometric units involved in the expression of such factors as brightness, illumination, speed, etc., it may be well to emphasize again that in formula (9) the brightness value must be in terms of *foot-lamberts* and the speed value in terms of exposure values expressed in *meter-candle-seconds*. If, now, values of B_0, S, and f be inserted and the value of t determined, this will be the time in seconds which will result in the rendition of the object brightness B_0 at a point on the curve where the gradient is $0.3\bar{G}$ of the average gradient of a 1.50 log exposure range on the characteristic curve. It has been shown[13,14] that if a scene be rendered with its lowest brightness, B_0min., at this gradient point, a negative will be produced from which an excellent print can be made. However, if the exposure is decreased progressively below this value, the prints made from the resulting negatives exhibit a rapid decrease in quality. In practice, therefore, it is desirable to introduce a safety factor into Eq. (9) which will give, on the average, appreciably more exposure than is called for by Eq. (9) as was shown above.

Equation (9) contains implicitly the relation between I_{40} and B_0. Hence in the form shown it is applicable only to a camera-lens system having zero flare. If flare light is present, a suitable term must be introduced in order to compute the exposure time which will result in the rendition of a given object area of known brightness, B_0, at a desired position on the characteristic curve of the negative material. This will be discussed in a later section.

E. Safety factor

For the successful application of Eq. (9) to practical work, it is necessary that the various factors involved be precisely known. In general, there may be some uncertainty as to the actual values of these factors under the existing conditions. For instance, there may be certain experimental errors in the estimation or measurement of object brightness values. The exposure time as determined from the shutter marking and the values of aperture ratio, f, may be subject to some uncertainty. The speed number used may not represent precisely the actual speed of the particular piece of film on which the negative is to be made. Furthermore, the amount of flare light present is not known precisely, nor can this be predetermined from the characteristics of the optical system (lens, camera shutter, diaphragms, etc.) since the value of flare light under any circumstances depends profoundly upon the brightness distribution within the scene being photographed and within the surrounding areas. The structure of this brightness distribution is so complicated that it is impossible, at least with our present knowledge, to predict from brightness measurements made on the scene and its surroundings the magnitude of the flare light.

In view of these possible uncertainties, it is wise, if not absolutely necessary, to introduce a safety factor which will insure the giving of sufficient exposure so that a negative from which an excellent print can be made will be obtained. Estimation of the probable uncertainty in each of the factors concerned may be made and hence a fairly intelligent decision can be made as to the magnitude of the safety factor which it is advisable to use. It is improbable in any particular

[13] L. A. Jones, "The evaluation of negative film speeds in terms of print quality," J. Frank. Inst. **227**, 297, 497 (1939).

[14] L. A. Jones and C. N. Nelson, "A study of various sensitometric criteria of negative film speeds," J. Opt. Soc. Am. **30**, 93 (1940).

case that the departures from the stated values of these factors will be all plus or minus and hence it seems unnecessary to adopt a safety factor sufficiently great to provide for such a contingency.

After a careful consideration of the possible errors that may be encountered, we have concluded that a safety factor of 2.5 should be adequate. Any value less than this might result in underexposed negatives in some cases. There seems to be little danger that a safety factor of 2.5 will result in overexposure. Reference to Table II shows that the average image illumination scale, IS_i, for the camera-lens system used in this work is 68. It is estimated that for the average equipment used in the amateur field the flare factor is appreciably greater than in the equipment used in this investigation. The evidence which we have at hand indicates that on the average the negative material is called upon to render an image illumination scale of about 40. Since the exposure scale of modern negative materials is great (500 to 1000) as compared with this value, there is little danger of encountering serious overexposure by using a safety factor of 2.5, as suggested above.

It should be remembered, however, that in the case of black-and-white reversal processes and in the case of color reversal processes, such as Kodachrome, the exposure scale of the material is much less than the exposure scales of the negative materials used in the black-and-white negative-positive process. In any exposure formula that is to be used for these reversal processes, the safety factor chosen must necessarily be considerably less than that mentioned as suitable for the negative-positive process.

F. Lens flare and flare factor

In choosing some particular object brightness as a criterion for the computation of the exposure required to give a successful result, it seems most rational to choose the *minimum object brightness*, B_0min. and to endeavor to determine the exposure conditions which will result in the rendition of this object area at the point on the characteristic curve where the gradient is equal to $0.3\bar{G}$. If there were no flare light present in the camera-lens system, this result could be accomplished by applying formula (9) and inserting a safety factor of 2.5 in order to avoid danger of underexposure because of uncertainty in the assumed values of the various factors involved. Since under average conditions a certain amount of flare light is always present, some correction must be made for this condition.

It is necessary now to decide how flare may be expressed quantitatively in a manner convenient for introduction into the final exposure formula. The flare characteristics of photographic objectives have been studied carefully by several investigators in the photographic field. Goldberg[5] gave this subject much attention. He developed a method whereby the flare characteristics of a lens could be specified numerically. This he accomplished by measuring the amount of light scattered into the image of a blackbody (one having zero brightness) when this small blackbody was surrounded by a large brightly illuminated field. On the basis of these measurements he was able to express the "specific brilliancy" of a lens system. He also considered the role of the camera and its various elements, the shutter and diaphragm blades, and lens barrel, in contributing to the total nonimage-forming light on the focal plane and concluded that flare light due to these elements was of negligible magnitude. A large number of measurements made by us indicate that, with many modern cameras, particularly of the small compact miniature type fitted with wide-aperture lenses, the contribution to the total nonimage-forming light on the focal plane by elements of the camera and lens mounts is far from negligible. There seems to be no possibility of using the flare value for a lens as such (the "specific brilliance" of Goldberg) to determine the magnitude of the nonimage-forming light on the focal plane when that lens and its associated camera assembly is used in different environments. This is evident from an examination of the flare factor values shown in Tables I and II. All of these scenes were photographed with the same lens-camera assembly. The "specific brilliance" of this lens, as measured according to the Goldberg technique, is approximately 50. It will be seen that the flare factors vary enormously from scene to scene. This variation is due largely to the variable brightness distribution found in these scenes and in their environments. From the summary of the entire

group at the bottom of Table II, it will be seen that the flare factors vary from 1.15 to 9.50.

An inspection of Table II shows that the average flare factor for group 1 is 1.90, which increases progressively through the first four groups to a value of 2.55 for group 4. The average brightness scale also increases progressively through these four groups and, as a matter of fact, there is a very good correlation between the average flare factors and the average brightness scales. The correlation between flare factor and brightness scale of individual scenes is not particularly high but nevertheless is indicated.

The average flare factor for the entire group is 2.35 which for further purposes we shall round off to 2.4 since the second decimal is probably not significant.

An extensive investigation of the flare characteristics of other lenses and cameras, particularly of the small compact miniature type, is now approaching completion and the results will be presented in a later report. The evidence at hand indicates definitely that the average flare factor for the average small camera used in large numbers in the amateur field is appreciably higher than that for the equipment used in this investigation. An average of a large number of measurements indicates a value of approximately 4.0, $\log FF = 0.60$. While additional evidence may change this value slightly, it is felt that the adoption of 4.0 is sufficiently near the statistical average to be satisfactory for use in the problem of computing camera exposures in the amateur field.

G. Practical exposure-time formula

By introducing the safety factor and the flare factor into Eq. (9) we obtain the exposure-time formula in its final form:

$$t = 4f^2 \cdot SF/5B_0\text{min.} \cdot S \cdot FF. \qquad (10)$$

Assuming the following values

$$FF = 4.0,$$
$$SF = 2.5,$$

the expression (10) reduces to

$$t = f^2/2B_0\text{min.} \cdot S. \qquad (11)$$

It should be remembered that this evaluation of exposure time is based upon assumptions as to the most probable average values of several factors as they apply to the average conditions which we believe to exist, especially in the field of amateur photography. We feel that this evaluation represents the best possible approximation to an exposure-time formula applicable to the statistical average conditions. These averages apply to the equipment used predominantly in this field. The result of using this formula is illustrated in Fig. 11. The solid curve *CDA* represents the sensitometrically determined density-log exposure characteristic of a typical negative material. The exposure corresponding to the point *C* determines the gradient speed of the material. The use of an exposure time as determined by Eq. (11) will render the minimum object brightness at the point *D*. The gradient at this point is 0.56. The fractional gradient is $0.70\ \phi_z$, which is appreciably greater than the limiting gradient required for the production of a negative from which an excellent print can be made. Assuming that the scene being photographed has a brightness scale of 160 and that the flare factor is 4.0 (which we consider to be the average obtainable under exterior conditions with the average small hand camera), the illumination scale of the image incident on the negative material is 40 and is rendered on the portion of the sensitometric curve between the points *D* and *A*. The dotted curve *BA* can be constructed by the method explained in connection with Fig. 4. This is the log B_0-D_n characteristic of the *negative*. The gradient of this characteristic at the point *B* is one-fourth of the gradient of the sensitometric characteristic at point *D*, being in this case 0.14 and the fractional gradient $0.24\ \phi_n$. The value is well above the limiting gradient required for a negative from which an excellent print can be made.

Referring to Table I, it will be seen that the ratio of the maximum to the minimum B_0min. values for the group 3 scenes is 5.5. It is evident, therefore, that the safety factor of 2.5 which has been incorporated in Eq. (11) is ample to take care of the inhomogeneity of brightness level within this group of scenes. Thus if the exposure time be computed using the average value of B_0min., that is 12.5 foot-lamberts, the use of this exposure time should give perfectly satisfactory negatives for all of the scenes within this group.

An inspection of Table II shows that the homogeneity of the first four groups is of the same order as that of group 3. Hence if by a method of inspection or measurement scenes can be successfully classified with the same degree of homogeneity as that obtained in Table II, average B_0min. values for each group should give a satisfactory determination of exposure for all scenes within each group.

H. Exposure time required for various scene groups

Considering further the data in Table II, the first three groups represent the order of homogeneity of illumination characteristics which can be obtained by classification according to the system discussed earlier in this paper. All of these groups fall under the general illumination condition of sunlit, No. 1 being for distant, No. 2 for remote, and No. 3 for nearby scenes. In the case of group 3 the local illumination condition is that designated as *unshaded*. Into group 4 were placed all scenes classifiable under general illumination as sunlit, but with the object of chief interest shaded from direct sunlight. A careful inspection of these scenes indicates that some should be grouped under the local illumination condition designated as *light shade*, while others fall in the *heavy shade* category. By splitting group 4 into two subgroups, on the basis of light or heavy shade conditions, still greater homogeneity of brightness characteristics can be obtained. This was done and an average B_0min. value of 6 foot-lamberts was obtained for the light shade subgroup and 4 foot-lamberts for the heavy shade subgroup. In the first section of Table III, indicated by the symbol I–1 at the left, values of average minimum brightness for these five subdivisions are shown in the line B_0min. It is interesting to note that these values decrease very systematically by a factor of 2 between consecutive members, especially in the case of the first four members of the series. The value for the fifth member, heavy shade, actually obtained was 4, but since only a few scenes fell within this group we consider it justifiable to alter this number to 3 making it fit into the decrement rate established by the first four members. The same general relation between the measured B_0min. values in the second, third, and fourth sections of this table is also indicated, although one or two values depart appreciably.

Group 5 consists of scenes which fall within several of our classification groups, such as haze, light clouds, heavy clouds, etc. The number of scenes falling in any one of these groups is hardly sufficient to establish with certainty a reliable average for the B_0min. value. However the data available, although somewhat fragmentary, are indicative of existing conditions. In the second, third, and fourth sections of Table III, indicated by the captions at the left, I–2, I–3, and I–4, have been entered the values of average B_0min. for the various subdivisions under each of these general illumination conditions.

In the first part of our classification outline are shown relative values of illumination levels existing for the four general illumination conditions. These are based on some measurements made by these Laboratories but largely on a study of the literature relating to brightness levels under various exterior conditions. If now we assume that the relative illumination levels are as indicated, we may enter in the second line of each section of this table the predicted values of average B_0min. for the groups I–2, I–3, and I–4.

Using Eq. (11) and assuming a film speed of 125 (Eastman Panatomic-X, Agfa Ansco Plenachrome, du Pont Superior, etc.) and an f value of 11.3, formula (11) reduces to the rather simple expression

$$t = 1/2 \cdot B_0\text{min.}$$

In the third line of each section of Table III, designated as t_c, are shown the required exposure values as computed. In the fourth line of each section, designated at the left by t_r, are the exposure times as recommended in various exposure guides and exposure tables published by the Eastman Kodak Company for photographic materials having a speed of 125. In these guides the scene types and illumination conditions are established by verbal description, accompanied in some cases by pictorial reproduction of typical scenes and by pictoral representations of illumination conditions.

It will be seen that the computed exposure times for the sunlit scenes agree almost perfectly with those recommended. The same very satis-

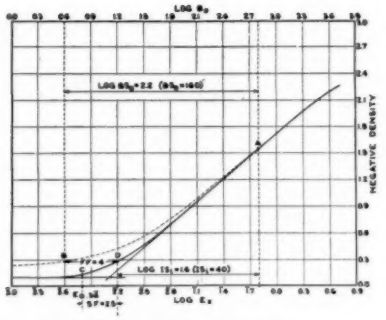

FIG. 11. Sensitometric curves illustrating the consequences of applying the exposure-time formula (11).

factory agreement between the computed and recommended exposure times is found for the subjects under haze. In the case of subjects illuminated by light clouds, two of the values give excellent agreement, while two of the computed values are approximately one-half as great as those recommended. In the case of scenes under heavy cloud illumination, only two of the groups are represented by the scenes which we have studied, but in these cases the computed exposures agree quite well with those recommended.

The number of scenes in each of the five subdivisions under sunlit illumination is large enough so that we feel the statistical average is of definite significance. The agreement throughout the remainder of the table is so good as to be very suggestive, if not conclusively significant. The authors of this paper feel confident that, when a sufficient number of measurements have been made on scenes in each of these categories, the average B_0min. values will be found to agree quite well with the numbers in parentheses which represent the stepping down from the high illumination levels to the lower ones in accordance with the values given earlier in this paper.

VII. Negative Characteristics

In Table IV are given some values relative to the negatives of the scenes classified as group 3. The first two columns show the minimum and maximum densities, while the third gives the density scale of the negative. These values, it will be remembered, are used in the determination of the flare factor and the image illumination scale as discussed previously. In the column designated as $h\theta_x$ are given the limiting gradient values of the used portion of the *negative material* and since flare light in the camera is usually negligible as compared with the maximum image illumination, this value is practically identical with the limiting gradient of the *negative* at the highlight end of the tonal scale. It will be noted that these values vary but little from an average of 0.77 which is approximately equal to the gamma to which these negatives were developed. In the case of one or two high values, the negatives were made on Panatomic Film and were processed to a somewhat higher contrast than those made on Super Sensitive Panchromatic Film. In a few cases a relatively low value of this quantity is found, and this is due to a rather excessive exposure which pushed the highlight density onto the beginning of the shoulder part of the negative characteristic.

The limiting gradient values of the used part of the negative material at the shadow end are in the column $s\theta_x$. It will be noted that the average is 0.46.

In this work no attempt was made to use the minimum exposure which would result in a negative from which a print of excellent quality could be obtained. On the contrary, an effort was made to give sufficient exposure so that the minimum density would be high enough for precise measurement, thus increasing the reliability of calculations involving this factor. While the exposure used was in most cases greater than necessary, it is evident from an inspection of Table IV that even those scenes having the highest brightness scale values have not suffered from loss of detail in the highlight regions resulting from overexposure. This is due to the extensive latitude of the film and to the lens flare being of such magnitude as to reduce the image illumination range to a sufficiently low value. In the last column, $s\theta_n$, are given the limiting gradients at the shadow end of the tonal scale of the *negative* characteristics, curve 2, Fig. 4. These gradients are the values of importance in determining the quality of detail rendering in the final reproduction. When they are multiplied by the limiting gradient of the used portion of the positive material, they give the limiting gradient of the reproduction.

In Table V are given the summarized values of the negative data for the other four scene groups.

These correspond to the summaries of scene characteristics shown in Table II. The average density scale increases progressively from group to group, which is to be expected since the average log BS_0 values increase from group to group, and the processing for all negatives was constant. Considering the various values of limiting gradient, there seems to be no indication of systematic relationship between either individual or average values and scene types. Thus the range of variation in $s\theta_n$ within any group is rather great and of about the same order in all five groups. It is only when these limiting gradient values are combined with the limiting gradients of the used portion of the positive materials that certain very definite systematic relations begin to emerge. This subject cannot be treated in detail in this communication but will be reserved for one soon to follow which will deal with the relation between characteristics of negatives and of positive materials which result in excellent tone reproduction quality in the final positives.

VIII. Calculator Numbers

The practical exposure-time formula has been reduced to a relatively simple form (11). It will be noted that the term B_0 appears in this expression and in the most general case this may refer to any selected brightness value in the scene to be photographed. We have already expressed our conviction that of the various possible brightness characteristics of the scene which may be measured or estimated, the minimum brightness, B_0min., is the most fundamentally rational and significant criterion of correct exposure.

Assuming then that we have for a given scene the value of B_0min., we shall now consider the problem of how the photographer can determine the value of t, together with a suitable value of f, in the simplest and most direct manner. It is obvious that if the speed of a photographic material is known it may be substituted directly in Eq. (11) and thus give the value of t for any desired value of f. It is relatively simple to make calculators of the slide rule type which may be put on an exposure guide or exposure meter to solve this equation.

The expression of speed as defined previously in this communication results, for modern medium- and high-speed negative materials, in numbers having three digits. It has been contended that these speed numbers are inconvenient since they produce crowded printing on mechanical calculators where it is desired to make these devices relatively small and attachable to exposure meters or guides that can be carried conveniently in the pocket.

There is, however, a more cogent argument for the establishment of a series of numbers designated by a name other than *speed numbers*.

There should be no possibility of ambiguity in the meaning attached to the word *speed* as applied to a photographic material. For several decades this word has been used as descriptive of a characteristic of the photographic material itself. It is extremely desirable to maintain in the literature of photography a very clear, distinct understanding that the word speed, as applied to a photographic material, shall retain this connotation. Nothing should be done that will lead to the inclusion, within the speed concept, of the characteristics of other elements of a complete photographic system. We should exercise the greatest care in an effort to reserve the word *speed* for use in specifying a characteristic of a photographic material without the inclusion of those other factors which must be taken into account in arriving at the correct exposure time when using optical systems of known characteristics, various methods for estimating or measuring the brightness characteristics of objects, the application of necessary safety factors, etc.

Table III. *Comparison of exposure times computed by use of formula (11) with those which have been found by long experience to be satisfactory.*

Group			Distant	Remote	Unshaded	Near-by Light Shade	Near-by Heavy Shade
I–1	Sun	B_0min.	55	24.5	12.5	6	3
		t_e	—	—	—	—	—
		t_r	1/110	1/49	1/25	1/12	1/6
			1/100	1/50	1/25	1/12	1/6
I–2	Haze	B_0min.	22	16	7	3	1.4
			(27)	(12)	(6)	(3)	(1.5)
		t_e	1/44	1/32	1/14	1/6	1/2.8
		t_r	1/50	1/25	1/12	1/6	1/3
I–3	Light clouds	B_0min.	29	8	6	1.5	—
			(14)	(6)	(3)	(1.5)	(0.7)
		t_e	1/58	1/16	1/12	1/3	—
		t_r	1/25	1/12	1/6	1/3	1/1.5
I–4	Heavy clouds	B_0min.	—	2.0	1.0	—	—
			(5.5)	(2.4)	(1.2)	(0.6)	(0.4)
		t_e	—	1/4	1/2	—	—
		t_r	1/10	1/5	1/2	1/1	1/0.5

This desirable result can be accomplished by incorporating in a number, for which the most suitable name appears to be "calculator number," all of the various factors involved in the determination of the correct exposure time, with the exception of the brightness characteristic of the object.

Formula (11) may be written in the form:

$$t=\frac{1}{B_0\text{min.}}\cdot\frac{f^2}{2S}, \qquad (11')$$

in which the term $f^2/2S$, as it stands, appears to involve only f and S, but, as a matter of fact, contains many concealed factors. Thus far the development of the exposure-time formula has been a process of simplification. Let us now reverse this process and examine this expression in its completely unsimplified form:

$$t=\frac{1}{B_0\text{min.}}\cdot\left[\frac{1}{S}\cdot\frac{4f^2\cdot SF}{\left(\frac{u-F}{u}\right)^2\cdot\cos^4\theta\cdot H\cdot T_g\cdot FF\cdot 10.76}\right]. \qquad (12)$$

By using reasonable and/or most probable statistical average values for all of the factors appearing in (12) with the exception of f and S, it reduces to (11′). In this form it has a deceptive appearance of simplicity but it should be kept in mind that the second part of the right-hand member takes its final form only by virtue of the assumed values for many factors upon which the final value of exposure time must depend.

We shall now consider some of the possible procedures which may yield a series of calculator numbers that will be simple, convenient, and adaptable to the purpose for which they are intended.

TABLE IV. *Detailed data pertaining to the characteristics of the negatives of scenes included in group 3.*

Scene No.	D_nMIN.	D_nMAX.	DS_n	Log BS_0	$h\theta_x$	$s\theta_x$	FF	$s\theta_n$
7	0.21	1.88	1.67	2.24	1.00	0.34	1.25	0.27
8	.16	2.12	1.96	2.59	.89	.26	1.35	.19
18	.56	2.00	1.44	2.52	.86	.50	5.30	.09
27	.47	1.63	1.16	2.22	.81	.45	3.80	.12
31	.18	1.61	1.43	2.69	.86	.20	2.40	.08
32	.31	1.33	1.02	2.16	.73	.42	3.10	.14
34	.34	1.47	1.13	2.59	.73	.42	6.70	.06
39	.31	1.72	1.41	2.16	.86	.48	1.85	.26
45	.23	1.76	1.53	2.10	.95	.37	1.35	.27
60	.62	2.16	1.54	2.38	.65	.53	1.70	.31
72	.38	1.85	1.47	2.48	.75	.44	2.20	.20
73	.57	2.23	1.66	2.54	.62	.52	1.70	.31
75	.41	1.74	1.33	2.32	.79	.41	2.30	.18
76	.70	2.10	1.40	2.16	.67	.61	2.10	.29
77	.28	1.75	1.47	2.76	.77	.32	2.70	.12
78	.40	1.82	1.42	2.39	.76	.40	1.65	.24
79	.33	1.31	.98	2.13	.81	.36	2.65	.14
80	.38	1.37	.99	1.65	.80	.39	1.30	.30
81	.60	1.90	1.30	2.13	.71	.53	4.70	.11
83	.35	1.33	.98	1.86	.80	.38	2.10	.18
86	.39	1.72	1.33	2.20	.84	.41	2.15	.19
88	.56	1.77	1.21	1.84	.79	.52	1.45	.36
89	.41	1.52	1.11	2.11	.80	.42	2.75	.15
90	.58	1.70	1.12	1.62	.80	.50	1.25	.40
98	.77	2.14	1.37	1.98	.59	.66	1.45	.46
116	.36	1.64	1.28	2.13	.77	.37	1.60	.23
119	.27	1.73	1.46	2.62	.70	.37	1.85	.20
134	.36	1.65	1.29	2.11	.71	.55	240	.23
142	.24	1.62	1.38	2.40	.73	.40	2.20	.18
151	.40	1.57	1.17	2.08	.76	.50	2.90	.17
153	.41	2.04	1.63	2.79	.75	.53	3.10	.17
154	.40	1.97	1.57	2.39	.73	.51	1.65	.31
159	.57	1.84	1.27	2.30	.77	.59	3.60	.16
163	.42	1.82	1.40	2.77	.73	.55	5.90	.09
165	.38	1.53	1.15	2.35	.72	.52	4.80	.11
167	.42	1.60	1.18	2.04	.73	.55	2.25	.24
174	.45	1.88	1.43	2.53	.77	.54	3.40	.16
175	.37	1.55	1.18	2.24	.75	.48	3.40	.14
Min.	.16	1.31	.98	1.62	.59	.20	1.25	.06
Max.	.77	2.23	1.96	2.79	1.00	.66	6.70	.46
Av.	.41	1.75	1.34	2.28	.77	.46	2.40	.21

A. Case No. 1

The most direct procedure in the establishment of a series of calculator numbers, the various members of which apply to some particular value of S, is to define such calculator numbers as equivalent in value to the second part of the right-hand member of Eq. (11′). Thus:

$$N_1=\frac{f^2}{2S}=\frac{1}{S}\cdot\frac{4f^2\cdot SF}{\left(\frac{u-F}{u}\right)^2\cdot\cos^4\theta\cdot H\cdot T_g\cdot FF\cdot 10.76}.$$

To evaluate N_1 for any value of S, it is necessary to assume some value of f. Let us choose arbitrarily $f=8$. Then,

$$N_1=32/S. \qquad (13)$$

In Table VI are shown the values of N_1 for various speed numbers.

N_1 may now be defined as follows: *N_1 is the number into which the value of B_0min. must be divided to obtain the correct exposure time (in seconds) at $f/8$.*

An inspection of Eq. (13) might lead one to conclude that N_1 is a unique function of S. While it is true that it is inversely proportional to S,

we must not lose sight of the fact that it is also inversely proportional to many other factors. This is emphasized by an inspection of (12). Thus, N_1 is obviously inversely proportional to $\cos^4\theta$. Furthermore, it is inversely proportional to T_g, the effective transmission of the lens, also to the flare factor, and is directly proportional to the safety factor and to f^2. Thus, a change in any one of several factors would modify the relation as shown in (13). For instance, if, instead of assuming that a lens, because of reflection losses and absorption within the glass, transmits only 70 percent of the incident light, it be assumed that it transmits all the light, then (13) would take the form:

$$N_1 = f^2/2.8S.$$

The calculator number N_1 for use with a photographic material having a speed, let us say, of 160, would be 0.14 instead of 0.20 as given by Eq. (13).

Referring again to Table VI, it will be noted that these calculator numbers, N_1, are inversely proportional to the speed values. The use of a series of calculator numbers which are inversely proportional to the speed numbers has some definite advantages. There is a danger that a series of calculator numbers which are directly proportional to speed numbers and not differing widely in magnitude will tend to be confused with the speed numbers themselves and, in practice, may generally be referred to as speed numbers. This condition is undesirable. It seems preferable, if possible, to distinguish clearly between speed numbers and calculator numbers. The adoption of an inverse series of calculator numbers would certainly tend to produce this result.

The actual magnitudes of the numbers shown under N_1 in Table VI may not be particularly convenient for the average individual since they are in decimal form. If, instead of selecting $f/8$ as the diaphragm opening in terms of which to express these calculator numbers, we should choose $f/64$, the numbers then will be as shown in the column under the designation N_2.

$$N_2 = 2048/S. \tag{14}$$

The definition of these calculator numbers would then read as follows: *N_2 is the number into which B_0min. must be divided to obtain the correct exposure time at $f/64$.*

TABLE V. *Summarized average values pertaining to the negatives of all groups.*

	D_nMIN.	D_nMAX.	DS_n	Log BS_0	$h\theta_x$	$s\theta_x$	FF	$s\theta_n$
Summary of group 1 scenes								
Min.	0.21	0.95	0.68	1.43	0.63	0.35	1.15	0.11
Max.	.80	2.10	1.37	2.45	.86	.72	3.30	.53
Av.	.46	1.60	1.14	1.91	.74	.51	1.90	.28
Summary of group 2 scenes								
Min.	.21	1.18	.85	1.54	.65	.26	1.30	.10
Max.	.80	2.12	1.82	2.53	.93	.67	4.80	.44
Av.	.41	1.62	1.21	2.11	.75	.49	2.30	.22
Summary of group 3 scenes								
Min.	.16	1.31	.98	1.62	.59	.20	1.25	.06
Max.	.77	2.23	1.96	2.79	1.00	.66	6.70	.46
Av.	.41	1.75	1.34	2.28	.77	.46	2.40	.21
Summary of group 4 scenes								
Min.	.16	1.52	1.11	2.06	.52	.20	1.25	.06
Max.	1.05	2.69	1.96	2.88	.96	.78	8.70	.53
Av.	.53	2.05	1.52	2.54	.70	.50	2.55	.22
Summary of group 5 scenes								
Min.	.11	1.16	.85	1.43	.55	.21	1.15	.05
Max.	1.08	2.90	2.23	2.81	1.00	.80	9.50	.68
Av.	.59	1.88	1.29	2.22	.76	.56	2.75	.24
Summary of all scenes								
Min.	.11	.95	.68	1.43	.52	.20	1.15	.05
Max.	1.08	2.90	2.23	2.88	1.00	.80	9.50	.68
Av.	.47	1.76	1.29	2.20	.75	.50	2.35	.23

B. Case No. 2

While there is much to be said for establishing the calculator numbers on a scale inversely related to the speed numbers, there are some arguments against such a procedure. Let us consider therefore what possibilities may be found for a simple and convenient calculator-number series in which the numbers are proportional to the speed of the photographic material. This can be accomplished by formulating the calculator number series designated as N_3, in which N_3 is the reciprocal of the second term of the right-hand member of Eq. (12). Thus:

$$N_3 = S \cdot \frac{\left(\frac{u-F}{u}\right)^2 \cdot \cos^4\theta \cdot H \cdot T_g \cdot FF \cdot 10{:}76}{4f^2 \cdot SF}. \tag{15}$$

Assuming again a value of $f/8$ and the same values which we have previously used for the other factors appearing in (15), this expression reduces to

$$N_3 = S/32.$$

t is now given by the expression

$$t = \frac{1}{B_0\text{min.}} \cdot \frac{1}{N_3}. \tag{16}$$

TABLE VI. *Two possible series of calculator numbers which are inversely proportional to the speed numbers of negative materials. Calculator numbers rounded off for convenient practical use.*

SPEED	$N_1=\frac{32}{S}$ $f/8$	$N_2=\frac{2048}{S}$ $f/64$
1250	0.025	1.6
1000	.032	2.0
800	.040	2.5
640	.050	3.2
500	.064	4.0
400	.080	5.0
320	.100	6.4
250	.125	8.0
200	.16	10.0
160	.20	12.5
125	.25	16
100	.32	20
80	.40	25
64	.50	32
50	.64	40
40	.80	50
32	1.00	64
25	1.25	80
20	1.6	100
16	2.0	125
12.5	2.5	160
10	3.2	200

N_3 is the number by which B_0min. must be multiplied in order to obtain the fractional part of a second required at $f/8$.

The series of calculator numbers resulting from this procedure is shown under N_3 in Table VII. In this series the calculator number corresponding to a speed of 1250 is 40. This is sufficiently large so that it may be confused with the speed numbers.

If it seems desirable to make these numbers smaller in magnitude, this can easily be accomplished by adopting $f/11.3$ instead of $f/8$ as the standard aperture ratio in terms of which to express the calculator numbers. Under these conditions the new calculator number is given by

$$N_4=S/64.$$

The values resulting from this procedure are shown in the third column of Table VII. Either one of these series appears to be satisfactory. The authors feel that a little preference should be given to the use of $f/11.3$ as the standard aperture ratio because of the lower absolute values of the calculator numbers resulting therefrom and less likelihood of confusing them with speed numbers.

C. Case No. 3

It has been suggested that an even simpler and more significant definition of the calculator number might be constructed by basing it upon the exposure time and f value required to give the correct exposure for some well-known, easily describable, and easily recognizable scene type. For this purpose we might choose a scene which has a B_0min. equal to the average of those scenes for which the data are given in Table I. As stated earlier in our discussion of the classification system set up for assembling scenes into a few relatively homogeneous groups, this group (No. 3) represents most closely the vast majority of pictures which are taken by amateur photographers. The general illumination conditions for these scenes is described as sunlit. They were front-lighted and represent the spatial distribution described by the term, near-by. The local illumination condition is such that the objects of chief interest are unshaded. Referring to Table I it will be seen that the average B_0 min. for this group is 12.5 foot-lamberts. Solving Eq. (11) using this value and a film speed of 160 and 11.3 for f, we find:

$$t=1/31.25,$$

which, for all practical purposes, may be rounded off to

$$t=1/32.$$

On this basis, therefore, we may formulate the definition for another possible calculator number, N_5: *N_5 is the fractional part of a second required to give the correct exposure for an average scene (as defined above) at $f/11.3$.*

Expressing N_5 in terms of S, we have:

$$N_5=S/5.$$

The series of numbers resulting from this assumption are shown in the last column of Table VII.

If we wish to phrase the definition of this calculator number in a manner parallel to that used in defining N_3, it may be done as follows: *N_5 is the number by which B_0min. must be multiplied to obtain the fractional part of a second required at $f/3.16$.*

D. Exposure meter readings

The series of calculator numbers suggested in the previous sections have been based on the

assumption that the minimum brightness of the scene to be photographed, B_0min., is known. The question undoubtedly will be raised as to how the correct exposure can be determined when the only available brightness measurement on a scene is other than that of minimum brightness.

There are on the market a wide variety of exposure meters which measure the integrated brightness of a scene. An examination of these meters shows that in many cases the units in which they are calibrated are arbitrary, although some read in conventional photometric units, such as candles per square foot. In addition to the wide variation found in the scale markings on various types of integrating exposure meters, there is little uniformity in the relation between the meter reading and a specific brightness characteristic of the scene, such as minimum brightness, average brightness, etc. This variation from type to type results from certain characteristics of the meter structure, among which may be mentioned its acceptance angle.

Even in the case of some individual meter type, the relationship between its response and the minimum object brightness is exceedingly variable. There seems to be no possibility, therefore, that the reading of an integrating type exposure meter can yield a unique value of the required exposure time as defined in terms of the minimum object brightness. The best that can be done is to determine by a large number of observations the statistical or average relationship between a meter reading and minimum scene brightness. This can be done in spite of the fact that the scale readings of the meter may be in arbitrary units.

For any particular exposure meter it is possible to determine a number which may be referred to as the meter constant, M. Thus, suppose a reading of minimum brightness is made on a large number of scenes covering as completely as possible the various scene types which are met with in practice. Let the average of these minimum brightness values be designated as a. Suppose, further, that on each of this same group of scenes a reading is made with some exposure meter which we shall designate as z, and let the average of these meter readings be designated by b. The constant for this hypothetical meter, z, may then be defined as:

$$M_z = b/a.$$

Now, if on a particular scene a reading, R_z, is made with this exposure meter, the correct exposure time can be determined by using any one of our formulas for t. Thus, we might choose formula (16). The exposure time will then be given by

$$t = \left(\frac{1}{R_z/M_z}\right)\left(\frac{1}{N_3}\right).$$

Expressing this verbally, the meter reading, R_z, divided by the meter constant, M_z, multiplied by the calculator number N_3, gives the fractional part of a second required for correct exposure at $f/8$.

Since we have available considerable data obtained by using an exposure meter of the integrating type, we can illustrate this problem more concretely by showing a numerical example. The meter reading values shown in the fourth column of Table II are in terms of foot-lamberts, the average for the entire group of scenes studied by us being 640. Converting this back to the actual readings taken from the scale of the exposure meter as marked, this becomes 200. The average of B_0min. for this same group

TABLE VII. *Three possible series of calculator numbers which are directly proportional to the speed numbers of negative materials. Calculator numbers rounded off for convenient practical use.*

SPEED	$N_3 = \frac{S}{32}$ $f/8$	$N_4 = \frac{S}{64}$ $f/11.3$	$N_5 = \frac{S}{5}$ $f/3.16$
1250	40	20	250
1000	32	16	200
800	25	12.5	160
640	20	10.0	125
500	16	8.0	100
400	12.5	6.4	80
320	10.0	5.0	64
250	8.0	4.0	50
200	6.4	3.2	40
160	5.0	2.5	32
125	4.0	2.0	25
100	3.2	1.6	20
80	2.5	1.25	16
64	2.0	1.00	12.5
50	1.6	.80	10.0
40	1.25	.64	8.0
32	1.00	.50	6.4
25	.80	.40	5.0
20	.64	.32	4.0
16	.50	.25	3.2
12.5	.40	.20	2.5
10	.32	.16	2.0

is 14.5. The meter constant for this instrument, therefore, is given by

$$M_z = \frac{B_0(M)}{B_0\text{min.}} = \frac{200}{14.5} = 14.$$

If, now, this same exposure meter is used in making a measurement of the integrated brightness of a particular scene and if the instrument scale reading is 250, the value of R_z/M_z becomes 18.1, and the exposure time is given by

$$t = \frac{1}{18} \cdot \frac{1}{5} = \frac{1}{90}.$$

This is the correct exposure, at $f/8$, expressed as a fractional part of a second for a negative having a speed of 160.

It should be realized that the validity of the method for the determination of correct exposure in this manner is based upon the assumption that the integrated brightness as read by an exposure meter of this type bears a fixed relationship to the minimum brightness for scenes of all types.

Figure 2 shows that this assumption is far from valid, although regarded from a statistical point of view, it may be of sufficient validity to give satisfactory results in the majority of cases. For the entire group of scenes studied in this work, it will be seen that the integrated brightness as obtained by the exposure meter used was, on the average, 44 times that of the minimum brightness. But, as indicated by the small circles on each of the charted lines, there was considerable departure from this average in many cases.

IX. Summary

1. The *maximum* and *minimum brightness* of 126 exterior scenes of a wide variety of types has been measured. From these data it is found that the *average brightness scale* is 160.

2. Photographs of each of these scenes have been made under precisely controlled exposure and processing conditions. A camera-lens assembly in which a relatively small amount of nonimage-forming light (flare light) was incident on the negative material was used in making these negatives. From measurements made on these negatives and from sensitometric characteristics of the negative materials used it is found that the *average* value of *image illumination scale* is 68 and the *average* value of the *flare factor* is 2.4.

3. For the *average* type of small portable hand camera used predominantly in the amateur field, the *average* value of the *flare factor* is approximately 4.0. This value is based on a large mass of statistical evidence not published as a part of this communication. For the equipment used predominantly in the amateur field, therefore, the *average* value of *image illumination scale* is 40.

4. It has been shown that exterior scenes can be classified by inspection into a relatively few groups which are quite homogeneous with respect to brightness characteristics, especially with respect to *minimum brightness*. This classification is based upon a consideration of general illumination conditions, directional viewing aspect, spatial distribution of scene elements, and local illumination conditions within the scene.

5. A formula for the computation of correct exposure time based upon the *minimum brightness* of the scene has been derived. This formula takes into consideration the statistical *average* characteristics of cameras and lenses which are used predominantly in the amateur field. It also includes a factor to compensate for the effect of *flare light* and a *safety factor* of sufficient magnitude to insure avoidance of underexposure.

6. Exposure times as computed by this formula for typical scene groups based on the classification mentioned in 4 agree very well with those which have been found by long experience to give satisfactory results.

7. Three possible methods for the setting up of useful series of *calculator numbers* are discussed.

8. Measurements made with a photographic exposure meter of the integrating type on each of the scenes studied show that the ratio of the *integrated brightness* value thus obtained to the *minimum* or to the *maximum brightness* value varies enormously from scene to scene. For the particular integrating exposure meter used the *average ratio* of the *integrated* value to the *minimum* brightness was found to be 44. This permits the determination of a *meter constant* by means of which the *statistically* correct exposure can be predicted by using the exposure time formula already mentioned.

The Brewsterian Angle and Refractive Indices

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On the basis of Brewsterian angle measurements, a procedure is here outlined which yields refractive indices with uncertainties in the third decimal place. Since "bulk" refractive indices are desired, it is shown that the errors due to surface films may be eliminated by making measurements only on freshly ground and polished surfaces. Attention is directed to applicability of this procedure when the refractive indices of black materials are to be determined.

THIS work is the outgrowth of a lecture-demonstration which was developed to illustrate Fresnel's laws of reflection from isotropic media. In all treatises on advanced optics it is shown that if an electromagnetic wave of unit amplitude be allowed to fall on the plane surface of an isotropic medium at the angle of incidence i and angle of refraction r, the reflected amplitudes (electric vectors) $R_\perp$ and $R_\parallel$ for polarized light, vibrating, respectively, perpendicular and parallel to the plane of incidence, are given by the relation:[1]

$$R_\perp = -\frac{\sin(i-r)}{\sin(i+r)}, \quad R_\parallel = -\frac{\tan(i-r)}{\tan(i+r)}.$$

The change in phase of the $R_\parallel$ component in passing through the Brewsterian angle ($i+r=90°$) can be shown effectively by reflecting light from a horizontal sheet of black glass. The incident light is passed through a sheet of Polaroid so oriented that the direction of vibration makes an angle of 45° with the (vertical) plane of incidence. The diagrammatic representation of the azimuth of the restored plane polarized light R after reflection is presented in Fig. 1 from which it is evident that, in passing from $i=0°$ to $i=70°$ the vector R rotates continuously in the same direction—which can happen only if the vector $R_\parallel$ experiences a phase-change of π in passing through the Brewsterian angle.

For common materials such as glass, water, etc., the Brewsterian angle i_B is so ill-defined that its determination can hardly serve as a reliable basis for the evaluation of refractive index ($n=\tan i_B$). Upon reconsidering the arguments leading to Fig. 1, it was realized that the relatively small angular change in the direction of R between $i=50°$ and $i=70°$ could be enhanced greatly by allowing the incident plane polarized light to vibrate in a direction which makes an angle of only a few degrees with the plane of incidence. Under these conditions the angular change ϕ in R, upon passing through the Brewsterian angle, is large. This is illustrated diagrammatically in Fig. 2.

With these ideas in mind, the apparatus shown in Fig. 3 was assembled. Here S is an automobile headlight lamp rigidly attached to the collimator arm of a spectrometer. The slit is removed and is replaced by the positive lens L upon whose inner

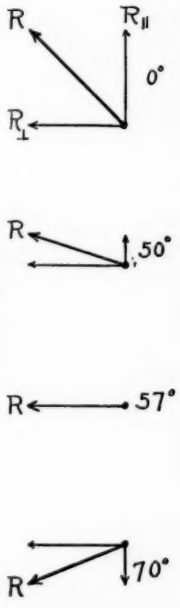

FIG. 1. R represents the resultant and azimuth of the restored plane-polarized light reflected from glass at the angles of incidence $i=0°$, 50°, 57° and 70°. The phase-change of the $R_\parallel$ component for $i=50°$ and $i=70°$ is brought out.

[1] *Handbuch der Physik*, Vol. 20, p. 211.

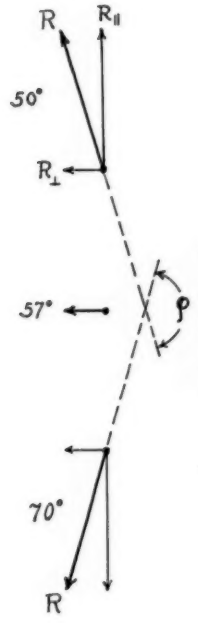

FIG. 2. The angle ϕ represents the large change in the azimuth of the restored plane-polarized light when the incident plane-polarized light makes only a small angle with the $R_{\parallel}$ direction.

surface a minute ink-spot is deposited at I. This spot is in the focus of the collimator lens C; however, the position of the lamp S is so chosen that a real image (3×10 mm) of the lamp-filament is formed in the axis of rotation of the spectrometer table. The incident light is polarized by means of a sheet of Polaroid at P which can be rotated in its own plane. The holder K for the specimen M is a brass plate bearing a central opening 6×12 mm mounted in the axis of rotation of the spectrometer table. The polished surface of the specimen M is held in contact with the rear surface of K by means of a rubber band. The front surface of K is heavily coated with flat black paint. In the focal plane of the telescope lens T is mounted a half-shade analyzer H–S[2] yielding two beams of light polarized at right angles to one another—each making an angle of 45° with respect to the (horizontal) plane of incidence. The two halves of the field meet in a vertical line. A dense-green filter F, mounted beyond the eyepiece E, yields a moderately monochromatic beam of wave-length 5600A. In addition the green filter makes use of that spectral region for which the older Polaroid is at its best.

[2] A. H. Pfund, J. Opt. Soc. Am. **26**, 453 (1936).

Determinations of the Brewsterian angle are made by mounting the polished surface of the specimen in the holder K; next the polarizer P is rotated until the direction of the transmitted vibration makes an angle of about 5° with the (horizontal) plane of incidence. Upon swinging the telescope into position so as to receive the reflected light (image of ink-spot lying symmetrically on the vertical line of demarcation between the two fields of the half-shade H–S) the entire field will be illuminated brightly. It will be found, as a rule, that the "half-shade" reveals a large contrast. The central table as well as the telescope are then swung into such a position that the brightness of the two areas of the half-shade appears the same. If now the polarizer be rotated still further until the light is almost extinguished, a difference in brightness again is observed. A second and final change in the angle of incidence is made until this contrast is also eliminated. Final angular readings on the spectrometer are taken only when the ink-spot is located centrally and when the photometric balance is retained upon rotating the polarizer through the "horizontal" position. The "zero" reading of the system is obtained by lifting out the holder K, and setting the image of the ink-spot on the central line of the half-shade.

To form some idea as to the precision of setting, a piece of plateglass ($n=1.515$) was mounted on the spectrometer-table and five independent settings were made. The average deviation from the mean value of the angle i_B was 1.5′ which meant an uncertainty in n of

TABLE I.

	i_B	n_B
Aged side	24° 17′	1.549
Fresh side	31° 23′	1.784

±0.0015. For materials of higher refractive index the errors are somewhat larger. Similar settings for a freshly ground and polished surface of dense flint glass ($n=1.78$) revealed an average deviation from the mean of 2.5′ for i_B which meant an uncertainty in n of ±0.003. For a polished crystal of ZnS (sphalerite $n=2.37$) the uncertainty in n is ±0.005. This is not due to a falling off in the precision of setting (average deviation

TABLE II.

MATERIAL	SIDE	REFRACTIVE INDICES (n) MINIMUM DEVIATION	ABBE REFR.	BREWSTERIAN n_B	COMMENTS
Dense flint prism	Aged	1.783	?	1.549	Surface films showed faint interference colors
	Fresh		too high	1.784	
Medium flint prism	Aged	1.613	1.612	1.548	No visible surface film
	Fresh		1.612	1.616	
Jamin Intfr. plate	Aged		1.479	1.492	No visible surface film
	Fresh		1.516	1.518	

from mean is 2.7′) but rather to the circumstance that the numerical value of tan i_B increases more rapidly as the angle increases. All of these results may be summed up in the statement that within the range $n=1.5-2.37$ it is possible to determine refractive indices for freshly ground and polished surfaces, on an average, to within ± 0.003.

As the work progressed, it became increasingly evident that the existence of surface films can produce a large change in the apparent refractive index. In the past, surface-films have been studied by Jamin,[3] Rayleigh,[4] Drude[5] and, more recently by Blodgett,[6] Vasicek[7] and Jones.[8] In the present work no importance is attached to the numerical values of n_B obtained for surfaces bearing surface-films—save, that the low values indicate the existence of surface-films. The true purpose of this study is to define the conditions under which refractive index measurements, carried out by reflecting light from the surface of a substance, will yield measurements which are characteristic of the bulk (interior) of the substance. To illustrate how greatly surface films may falsify results, the case is here cited of a dense flint glass prism at least forty years old. Brewsterian angles i_B were determined (1) on the old or "aged" side bearing the surface film, (2) on the "fresh" side with its newly ground and polished surface. The results are presented in Table I.

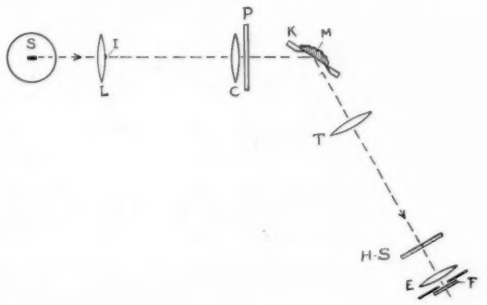

FIG. 3. Diagrammatic sketch of the apparatus used in determining refractive indices by means of the Brewsterian angle.

To establish, decisively, the conditions under which the measurement of i_B will lead to the bulk refractive index, this quantity was determined for two (40-year old) prisms by the usual minimum deviation method. Each prism had a "fresh" and an "aged" side. Surface films in the nature of plane-parallel plates would exert no influence on the final result. Next, measurements of n were made by means of an Abbe refractometer whose lower prism had been removed. Aroclor ($n=1.62$) was used as the liquid medium between the Abbe prism and the one to be tested. In addition an old Jamin interferometer plate was studied. Finally, Brewsterian angles, i_B and refractive indices n_B were determined, as shown in Table II.

It is evident that all three methods yield essentially the same value of n for the "fresh" side. Again, it appears that the values of n_B for the aged side are invariably the smaller. The Abbe refractometer fails to pick up the low refractive on the aged side of both prisms (due,

[3] J. Jamin, Ann. de chimie et de physique **29**, 263 (1850).
[4] Lord Rayleigh, *Collected Papers*.
[5] P. Drude, *Lehrbuch der Optik* (1900), p. 266.
[6] K. B. Blodgett, Phys. Rev. **55**, 391 (1939).
[7] A. Vasicek, Phys. Rev. **57**, 925 (1940).
[8] F. L. Jones and H. J. Homer, J. Opt. Soc. Am. **31**, 34 (1941).

TABLE III.

SUBSTANCE	n_B	n_D (AUTHORITATIVE)
Fluorite	1.433	1.434
Rocksalt	1.543	1.544
Quartz	1.545	1.549 (mean)
Barytes	1.643	1.640 (mean)
Zinc oxide	2.023	2.015 (mean)
Zinc sulfide	2.358	2.37
Methyl methacrylate	1.493	1.490
Black glass	1.520	
Anthracite coal	2.016	
Selenium (amorphous)	3.05	3.04 ($\lambda = 5600$A)

probably, to too slight a thickness of the surface-film). On the other hand, even the Abbe refractometer yields two different values for the two sides of the Jamin plate.* The conclusion to be drawn from these experiments is that both Abbe refractometer and Brewsterian angles will yield values of n characteristic of the bulk of the material only if measurements be carried out on "fresh" surfaces.

In the following Table III are presented values of n_B for a variety of crystals and other substances. Throughout, only "fresh" surfaces were used. For the most part the "authoritative" values of n_D obtained by dioptric methods were taken from the *Handbook of Physics and Chemistry* (Chemical Rubber Publishing Company), eighteenth edition.

Granting that the wave-lengths used in the evaluation of n_B are not quite the same as those used to obtain n_D, it is evident that no glaring discrepancies between the two sets of results appear. Two features of this work are to be emphasized: (1) the method, in contrast to the Abbe refractometer, has no upper limit; (2) reliable results may be obtained for black materials such as anthracite coal and amorphous selenium.

* While the line of demarcation in the Abbe was extremely sharp for the "fresh" side, it was rather ill-defined for the "aged" side and the upper portion of the field was colored non-uniformly. Such observations give warning that even the Abbe refractometer may yield incorrect results.

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A Quartz Photoelectric Spectrophotometer*

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ABSORPTION spectrophotometry has long been recognized as a rapid and versatile method for the identification and quantitative determination of many substances. Measurements in the ultraviolet with instruments commonly available usually have involved inconvenient photographic density determinations or the use of a delicate galvanometer when the photographic step is eliminated by the employment of a photo-cell or radiation thermopile. The speed and convenience of spectrophotometric determinations can be greatly increased by the use of a direct-reading potentiometer, electronic amplifier and rugged balance-indicating meter. The photoelectric spectrophotometer described in this paper incorporates these features for measurements in the ultraviolet and visible regions.

* Presented at the Ninth Summer Conference on Spectroscopy and Its Applications held at Massachusetts Institute of Technology, Cambridge, Massachusetts, July 21–23, 1941.

DESIGN FACTORS

For accurate photoelectric spectrophotometry, two fundamental requirements must be met. First, the sample must be illuminated with light of such spectral purity that the absorption as measured by the photoelectric device is sensibly that which would be obtained with ideally monochromatic light. Second, the monochromatic light must be available in such quantity that it can be measured to the desired accuracy with the photoelectric means employed. In the design of a photoelectric spectrophotometer a

choice must be made between various types of light source, monochromators and photoelectric devices so that these requirements will be economically and conveniently met.

As sources of continuous radiation for absorption measurements, the tungsten filament lamp for the visible and adjacent spectral regions and the hydrogen discharge tube for the ultraviolet are the most suitable. For the photoelectric measurements vacuum tubes, with appropriate electronic amplifiers, afford the greatest sensitivity and accuracy. The selection of a monochromator is not a simple matter, however, as the available literature does not provide adequate information to permit an intelligent choice to be made between the various types.

A major choice lies between a quartz prism and a grating as the dispersing element of the monochromator. The replica grating in particular is relatively inexpensive and it provides higher dispersion in the visible and infra-red than is easily obtainable with prisms, thereby permitting the use of wider slits and reducing the precision required in the construction of the slits and wave-length scale mechanism.

On the other hand, the grating has several defects in performance that offset these advantages. Chief among these is scattered light. It is very important that the monochromatic light illuminating the sample be free from stray light. In a well-designed monochromator, stray light originating in unwanted specular reflections from the optical elements and supporting members will be reduced to a negligible amount and most of the stray light will be due to non-specular scattering by the optical elements. Since the surface of a grating does not approach the optical perfection of the surface of a prism, it might be expected that the scattered light from a grating would be greater than that from a prism.

To obtain data to aid in selecting a monochromator design, tests were made on working models of five different types of monochromators. Two employed quartz prisms, one of which was a modification of the Fery self-collimating prism and the other a mirror-collimated Littrow type. The remaining three models employed gratings. Eight different gratings were tested, including an original concave reflection grating on speculum metal, an original aluminized plane reflection grating, two commercial concave reflecting replicas, two commercial transmission replicas and two aluminized plane reflection replicas made in our own laboratories.

An estimate of the percentage of scattered light present in the output of the various monochromators was obtained in the following manner. Using a tungsten filament source, the light leaving the exit slit of the monochromator was measured with a caesium oxide photo-tube. A filter which completely absorbed the monochromatic light but transmitted most of the scattered light was then inserted in front of the photo-tube and another reading taken. This latter reading, corrected for the estimated absorption of scattered light by the filter, was taken as a measure of the scattered light.

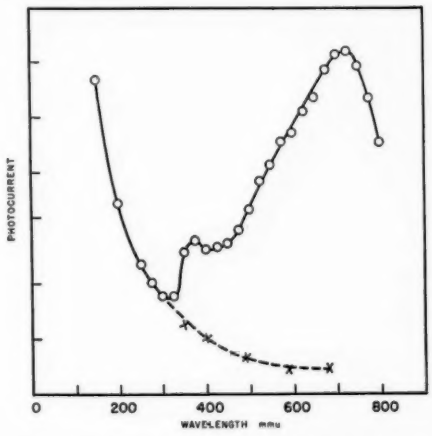

FIG. 1. Total and scattered light from grating.

Figure 1 summarizes the results obtained in the first-order spectrum of a concave reflection replica of 43-cm focus with 3000 lines per cm ruled in an area 35×48 mm. The slits were set for a nominal band width of 3 millimicrons. The points on the upper curve were taken without the filter and correspond to the total light received by the photo-tube. A striking feature of this curve is the rapid increase in light received by the photo-tube as the wave-length setting of the monochromator is decreased below 300 millimicrons. Inasmuch as the photo-tube had a lime glass envelope and was completely insensitive to radiation of these wave-lengths, it is

apparent that a rapid increase in scattered light occurs as the grating is turned to bring its undispersed beam near the exit slit.

The points on the lower curve correspond to the amount of scattered light as measured with various filters in front of the photo-tube. These points show the same trend noted above for the scattered light. No filters were available for making scattered light measurements at wave-lengths longer than 680 millimicrons but it may

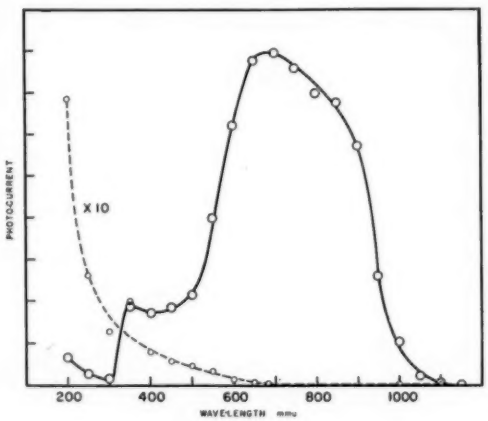

FIG. 2. Total and scattered light from quartz prism.

be surmised that no marked improvement in spectral purity occurs at longer wave-lengths since the second-order spectrum is encountered above 800 millimicrons.

The results given are typical of those obtained with other replica gratings. Two original gratings which were more efficient and therefore had a smaller amount of light in the undispersed beam, did not show as sharp a rise in scattered light at the lower wave-length settings. However, we were unable to find any grating for which the scattered light was less than 5 percent of the monochromatic light, as measured with a caesium oxide photo-tube.

Figure 2 shows data obtained with a quartz prism monochromator employing a thirty-degree aluminized prism autocollimated by a spherical mirror. At each wave-length setting, the slits were adjusted to a nominal band width of 3 millimicrons. In this case, the rise in the curve at wave-lengths below 300 millimicrons is due to increased dispersion of the prism at the shorter wave-lengths, which necessitates widening the slits to maintain the 3-millimicrons nominal band width. The amount of scattered light tends to increase in proportion to the product of the slit widths.

The scattered light at different wave-lengths, estimated in the same manner as with the gratings, is shown by the dotted curve of Fig. 2 on a scale magnified ten times. Between 650 and 1100 millimicrons, the amount of scattered light is so small as to be within the limit of experimental error and is not more than 0.1 percent of the monochromatic light. At 500 millimicrons the scattered light is about 2 percent of the monochromatic and this figure rises to about 7 percent at 350 millimicrons.

The data on scattered light obtained in the experiments outlined above would be greatly modified if the spectral sensitivity of the photo-tube were changed by using filters or if other types of photo-tubes were used. The relatively high red sensitivity of the caesium oxide photo-tube emphasizes the scattered light found at the shorter wave-length settings. Nevertheless, the data indicate that the quartz prism has a decisive advantage over the grating with respect to scattered light.

Optical System

The optical system chosen for the spectrophotometer is shown in Fig. 3. The monochromator is a modification of a mirror-collimated Littrow spectrograph and is capable of giving the desired performance with a minimum of optical and mechanical complications. The reflecting prism avoids the necessity of using right- and left-hand quartz and requires half as much quartz as the equivalent monochromator having a separate collimator and telescope. In addition, the use of a mirror as collimator avoids the expense of an achromatic lens or the focusing complications necessary with a simple quartz lens.

Two interchangeable light sources are used. For the spectral range 1000 to 320 millimicrons, a standard 32-candlepower automobile headlight lamp (Mazda No. 2331) is used. The current of 4.3 amperes required for the operation of the lamp is supplied preferably by a six-volt storage battery. In the spectral range 350 to 220 milli-

microns, a hot cathode hydrogen discharge tube is used. The discharge current of 0.4 ampere is obtained from a regulated rectified a.c. power supply. A description of the discharge tube and power supply is being prepared for future publication.

An image of the light source located at *A* is focused with five-fold magnification by condensing mirror *B* and diagonal mirror *C* on the plane of the slits at *D*. The condensing mirror has a radius of curvature of 5.7 cm and is aluminized on its front surface. Since it operates 25 degrees off-axis and at high numerical aperture, the image of the light source at the slit is highly astigmatic and distorted. Computation and experiment show that all rays passing through the entrance slit and striking the collimating mirror must originate in an area approximately 0.6 mm wide and 3.1 mm high in the vicinity of the source of *A*. Consequently, this condensing system enables a relatively small light source to provide all of the light that the monochromator is capable of taking.

At *D*, the entrance slit is located directly below the exit slit. Both slits are curved and have a working height of 13 mm. They are of the bilateral type, adjustable in width from 2.0 to 0.01 mm. The use of dovetail ways or similar devices for guiding the slit jaws in a parallel motion is avoided. The slit jaws are constrained to move in a plane by being lightly pressed in contact with a flat plate lying parallel with the plane of their motion. They are further constrained to move parallel with themselves in this plane by suspensions made of flat springs in the familiar parallelogram arrangement. A cam rotating in a conical precision bearing about an axis perpendicular to the plane of the slits is located between the slit jaws and presses them apart against the tension of the flat springs. The cam is rotated by a worm gear driven by the slit-adjusting knob. Because of the shape of the cam and the reduction of friction by the use of the flat springs, the motion of the slit jaws is very smooth and is fine enough so that the light flux through the slits may be adjusted by increments of 0.1 percent even when the slits are no more than 0.02 mm wide.

The collimator (*E*) is an aluminized spherical mirror of 50 cm, focal length, and is approximately 5 cm wide and 9 cm high. Since it operates about 5 degrees off-axis, it introduces appreciable spherical aberration and astigmatism. The aberrations act to widen the spectral band width of the light leaving the exit slit, the increase in band width being about the same as would be caused by increasing the slit widths by 0.02 mm. This increase in band width is negligible in the ultra-violet and blue since the instrument normally operates in this spectral region with relatively wide slits. In the infra-red, the aberrations limit the minimum spectral band width to about 2 millimicrons. By employing an off-axis parabolic mirror instead of the spherical mirror, a considerable improvement in performance could be obtained in the infra-red. However, since a band width of less than 5 millimicrons is rarely needed in measuring the infra-red absorption of solutions, the additional expense for a parabolic mirror is not justified.

The prism *F* has a 30-degree angle and is made from the best grade of crystal quartz. The back surface, which is aluminized, has a diameter of 5 cm. The prism is supported on bearings of kinematic design and is rotated by the wave-length scale mechanism.

The wave-length scale extends from 200 to 2000 millimicrons and is drawn in the form of a four-turn flat spiral 100 cm long. It is mounted on a disk 13 cm in diameter that is free to rotate on a conical precision bearing. The scale and its traveling index are viewed through a window on the top of the monochromator case. On the

FIG. 3. Optical system of spectrophotometer.

underside of the scale disk is cut an accurate four-turn scroll having a lead of approximately 1 cm per revolution. A long arm attached to the prism carrier bears a hardened steel pin that engages the scroll and causes the prism to rotate as the wave-length scale is turned.

PHOTO-TUBES

Two photo-tubes are used in this instrument. One is an RCA Type 919 caesium oxide photo-

tube similar to the photo-tube used in making the scattered light measurements. Since, as those measurements showed, this photo-tube is excessively responsive to scattered light at wave-lengths below 600 millimicrons, a special RCA C7032 photo-tube is used for measurements below 625 millimicrons.

The solid curve of Fig. 4 shows the photo-current of the special photo-tube, using the tungsten filament source and with the slits of the quartz monochromator adjusted for a nominal band width of 3 millimicrons. For purposes of comparison, data taken with the caesium oxide photo-tube (Type 919) under the same conditions are plotted on the same scale. These data show that the special photo-tube is insensitive to the longer wave-lengths but has extraordinary sensitivity to wave-lengths below 625 millimicrons. Since this photo-tube has an envelope of Corning 974 ultraviolet transmitting glass, it has high sensitivity to wave-lengths as short as 210 millimicrons and therefore is well adapted for work with a hydrogen discharge tube or other source of ultraviolet light.

When the tungsten lamp is used with the quartz monochromator, the scattered light measured by the special photo-tube is less than 0.1 percent of the monochromatic light between 625 and 425 millimicrons. Below 400 millimicrons, a Corning 986 red-purple Corex filter is placed in the light path and with this the scattered light is barely detectable at 350 millimicrons and amounts to about 0.8 percent at 320 millimicrons.

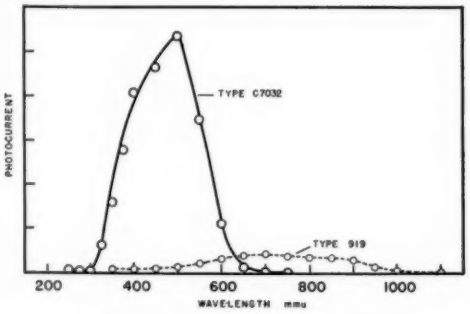

FIG. 4. Comparison of photo-tubes.

When the hydrogen discharge tube is used, the scattered light with the special photo-tube is negligible from 350 to 275 millimicrons, amounts to about 0.3 percent at 250 millimicrons and about 1 percent at 220 millimicrons.

ELECTRICAL CIRCUITS

The electrical circuits associated with the photo-tubes are shown in Fig. 5. The photo-tube current is measured by balancing the voltage drop developed in the 2000 megohm resistor (R_{10}) with the slide wire potentiometer (P_1). Several circuit features are included to avoid photo-tube dark current corrections and arithmetical calculations of the percentage transmission.

Balance between the voltage drop in the 2000-megohm resistor and the slide wire voltage is indicated on the milliammeter through the action of the amplifier. The amplifier employs a Type 32E electrometer tube and a Type 30 amplifier tube. It has an input resistance of about 10^{13} ohms and a grid current of about 2×10^{-13} ampere and will detect a voltage unbalance of 0.2 millivolt. In conjunction with the 2000-megohm resistor these characteristics lead to a photo-tube current sensitivity of about 10^{-13} ampere. The atmosphere in the box containing the amplifier and photo-tubes is maintained at a low humidity by a drying agent to avoid insulation leakage difficulties.

Switch S_1, which selects one or the other of the photo-tubes, is coupled with the slide mechanism that moves the photo-tubes into position, so that the necessary circuit changes take place automatically when changing from one photo-tube to another.

Switch S_2 is coupled with a shutter which, in the "off" position, prevents light from reaching the photo-tubes and is used for dark current compensation. By placing S_2 in the "off" position and adjusting potentiometer P_2 until the amplifier indicates balance, the dark current of the photo-tube is compensated so readings made on the slide wire potentiometer P_1 need not be corrected for dark current.

Switch S_3 has three positions. In the "check" position, the instrument is adjusted with sensitivity control P_3 so that a zero density reading is obtained when the absorption cell containing the blank is in the light path. In making this adjustment it is unnecessary to set the density scale

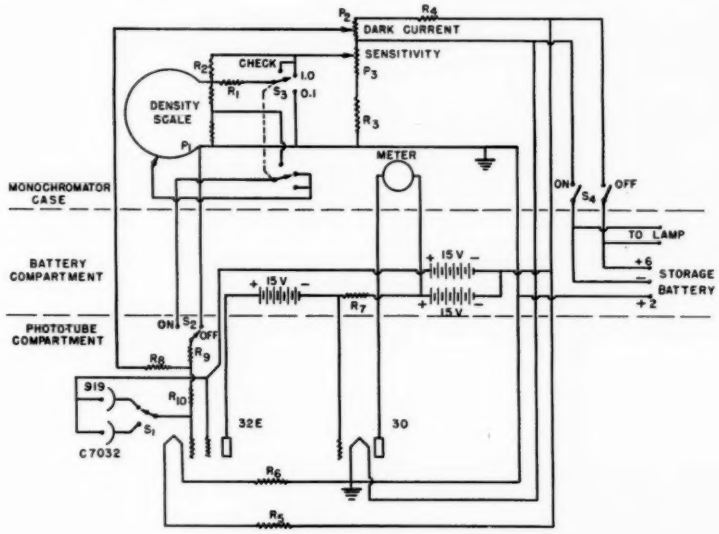

FIG. 5. Electrical circuits.

to read zero. Potentiometer P_3 has a nine-turn helical winding which gives a 10-fold variation in scale sensitivity and at the same time permits settings to be made by increments of 0.1 percent or less.

Density readings ordinarily are made with switch S_3 in the "1.0" position. Greater accuracy with samples having less than 10 percent transmission is obtained by placing switch S_3 in the "0.1" position, where the scale sensitivity is increased by a factor of ten. This switch position is also useful in allowing full scale readings to be obtained when working with very narrow spectral band widths. Of course, in this case the accuracy of the measurement is limited by the stability of the amplifier and photo-tubes.

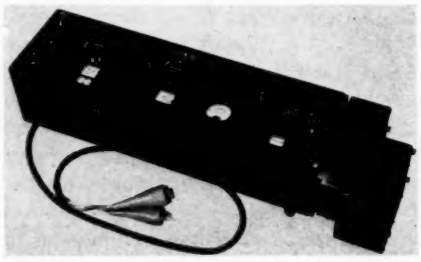

FIG. 6. Quartz photoelectric spectrophotometer.

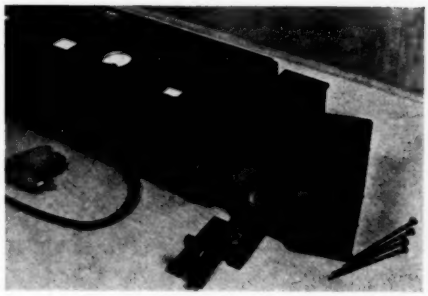

FIG. 7. View of spectrophotometer showing disassembled absorption cell and photo-tube compartment.

SPECTROPHOTOMETER CONSTRUCTION

The general appearance of the spectrophotometer is shown in Fig. 6. The over-all dimensions are: length, 30″; height, 8″; width, 9″. It weighs approximately 75 pounds.

The large cast aluminum case at the left houses the quartz monochromator. On its top surface are located the wave-length, slit width and density scales as well as the controls for the photo-tube and amplifier circuits. The optical elements of the monochromator are mounted in a rigid cast-iron box independently suspended inside the aluminum case. Crystal quartz windows over the

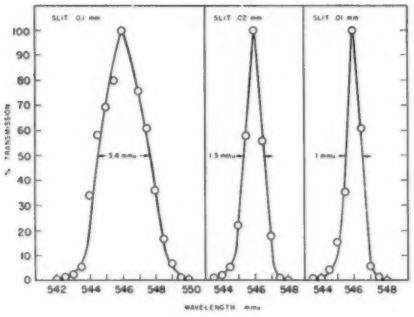

FIG. 8. Spectral isolation with various slit widths.

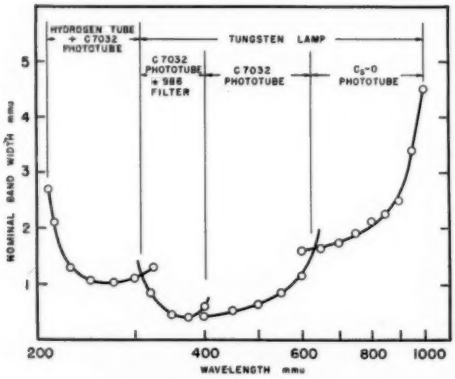

FIG. 9. Minimum spectral band widths which permit scale readings to 0.1 percent.

slits and gaskets at all joints prevent dust and fumes from reaching the optical system.

The entrance and exit slits are mounted on the machined plate at the right-hand end of the monochromator case. Interchangeable light sources, absorption cell compartments and photo-tube units are attached to this plate by thumb-screws and alignment pins. The absorption cell compartment illustrated holds four 10-mm square test tube type absorption cells in a slide holder. Absorption cell compartments taking cells with path lengths up to 10 cm may be inserted between the end plate and the photo-tube unit. The disassembled parts are shown in Fig. 7.

PERFORMANCE DATA

To test the performance of the optical and mechanical systems of the monochromator, a

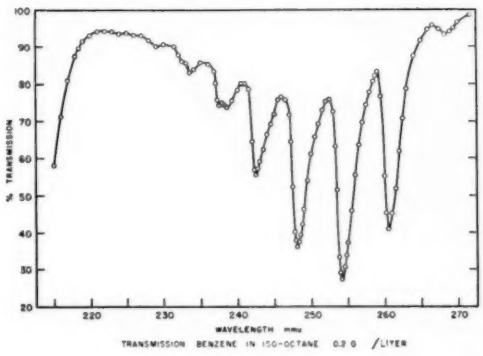

FIG. 10.

procedure used by Hogness[1] and co-workers was followed. With a mercury arc as light source, photo-tube readings were taken at a series of wave-length settings in the vicinity of several of the mercury lines. Figure 8 gives the results obtained at three different slit widths with the 546-m mu mercury line. It is to be noted that a reduction in the slit width does not cause a proportional reduction in the spectral region isolated. This is caused by the aberrations in the off-axis collimating mirror which, as mentioned previously, have the effect of widening the slits by about 0.02 mm so far as spectral purity is concerned. With this correction, the curves are in accord with the theory developed by Hogness and his co-workers. Similar results obtained at other mercury lines demonstrated that the wave-length scale was accurate and reproducible to better than 1 millimicron.

Figure 9 shows the minimum spectral band widths which give sufficient light for transmission readings to be made to 0.1 percent. The "nominal band width" figures given on the ordinates are the effective widths of the slits in millimicrons, allowance having been made for the aberration of the collimating mirror. Thus these figures give the "half intensity band width" as defined by Hogness. Narrower band widths than those indicated in the figure may be used, particularly in the blue and ultraviolet, but the reduction in light flux from the monochromator reduces the accuracy with which readings can be made.

[1] T. R. Hogness, F. P. Zscheile, Jr., and A. E. Sidwell, Jr., "Photoelectric spectrophotometry. An apparatus for the ultra-violet and visible spectral regions: its construction, calibration, and application to chemical problems," J. Phys. Chem. **41**, 379–415 (1937).

Figure 10, which shows the transmission curve for 0.02 percent benzene solution, is an interesting example of the work that may be done with this instrument. It is to be noted that all of the salient characteristics of the absorption of benzene are well defined.

ACKNOWLEDGMENT

We wish to express our appreciation to Mr. Douglas Marlow, who made many valuable contributions to the mechanical design of the instrument, and to Mr. Roger Hayward, who conducted the preliminary optical experiments.

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An Automatic Recorder of Spectral Sensitivity of Photoelectric Surfaces

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A method of recording automatically spectral sensitivity of photoelectric surfaces has been developed. It consists of a Hardy spectrophotometer supplied with attachments for 60-cycle modulation of two glow discharge lamps. One of the lamps, serving as light intensity reference source, is supplied with an iris whose aperture is controlled mechanically by the analyzer cam. The other lamp is controlled photoelectrically so that its light intensity is proportional to the relative spectral sensitivity of the investigated photoelectrical surface. Light emitted by both lamps is received by the detector photo-tube which by means of thyratrons controls the recording mechanism.

INTRODUCTION

DURING the study of new methods of sensitizing photoelectric surfaces the length of time necessary for taking spectral sensitivity curves of new surfaces by means of a monochromator, became prohibitive. A means was sought for avoiding the procedure of a point-by-point method of measuring the sensitivity for a large number of wave-lengths, reducing the values thus obtained to an equal energy scale, and then plotting the final spectral sensitivity curve for the investigated photoelectric cell.

Since a Hardy-type of spectrophotometer[1] made by the General Electric Company[2] for recording automatically the transmission and reflectance coefficients of different materials was available, the simplest approach to the solution appeared to be the following two steps. First, the Hardy instrument would have to be adapted for recording the spectral sensitivity of photoelectric surfaces. Secondly, means for obtaining equal energy for each wave-length setting would have to be provided. The description which follows deals with the first part of the task.

[1] Arthur C. Hardy, J. Opt. Soc. Am. **25**, 305 (1935).
[2] J. L. Michaelson, J. Opt. Soc. Am. **28**, 365 (1938).

PRINCIPLE

In the Hardy instrument both parts of the monochromatic beam are utilized directly within the integrating sphere, so that the detector photo-tube of the spectrophotometer receives the monochromatic light pulses which are gradually varying in wave-length. Instead, the photoelectric cell whose spectral sensitivity is to be investigated, is utilized for the translation of one monochromatic beam whose wave-length is gradually varying, into variations of light intensities emitted from a source of constant spectral distribution. This source is modulated sinusoidally at the rate of 60 cycles per second.

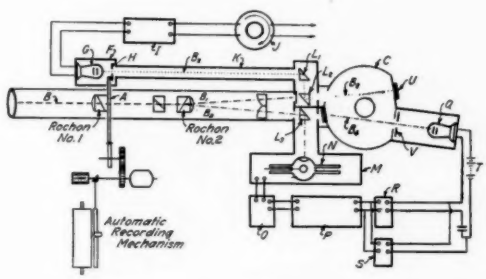

FIG. 1.

The amplitude of these sinusoidal light pulses varies proportionally to the spectral sensitivity of the investigated photoelectric surface.

The other beam acting as a reference source is replaced by one emitted from a second source of identical and constant spectral distribution but with its light pulses modulated 90° out of phase.

Description

The schematic diagram of the arrangement of the spectral sensitivity recorder is shown in Fig. 1. The parts belonging to the Hardy spectrophotometer are shown in light lines and represent the Rochon prism No. 1, the accompanying analyzer cam and cam rod A for controlling the beam intensity, the Wollaston prism, the revolving Rochon prism No. 2, the integrating sphere C, and the detector photo-tube indicated by a circle.

The adaptor for investigating the spectral sensitivity of the photoelectric surfaces is shown in heavier lines. The beam B_1 which for spectrophotometric measurements is used as a reference source, is not utilized and is blocked from entering the integrating sphere by means of a plate. This beam is replaced by another, B_2, which is emitted by the reference light source F. The latter consists of a neon lamp G and of a specially designed iris H whose aperture is varied by the cam rod A which also controls the angular position of the Rochon prism No. 1. The coupling of the iris with the cam rod is such that the same relative change in intensity of the beam B caused by the angular displacement of the Rochon prism No. 1 is translated mechanically to vary accordingly the aperture area of the iris. A constant ratio of intensities between the beams B and B_2 is established by this coupling.

Fig. 2.

The neon lamp G is fed from a power source I which supplies the d.c. component modulated by a 60-cycle a.c. so that 100 percent light modulation of the cathode of the neon lamp is obtained, the anode always remaining dark.

The ratio of the current components of the a.c. to d.c. is adjusted to remain constant for any voltage setting of the Variac J. The light beam B_2, passing through the iris is conducted through a tube K coated on the inside with magnesium oxide and directed into the integrating sphere by means of the prisms L_1 and L_2. The source F thus supplies the detector photo-tube with a pulsating beam of neon light varying at the rate of 60 cycles per second and serving as a reference beam.

The beam B_3 coming from the spectrophotometer is deviated by means of a reflecting prism L_3 into a compartment M, containing the investigated photo-tube N. By subjecting its photoelectric surface to the light pulses coming from the monochromator, electrical current pulses are produced which are raised in amplitude first by means of a pre-amplifier 0, using a General Electric FP 85 space-charged grid electron tube, and a 15-watt voltage amplifier P. The output of the amplifier P feeds into two channels. One channel supplies the alternating component of the photoelectrically controlled neon source Q through a coupling transformer R. The other channel leads to a rectifier S and supplies the direct-current component for the neon lamp Q. An additional biasing voltage is applied by a battery T which raises the voltage of the cathode of the neon lamp to a level slightly above the breakdown potential of the lamp. The circuit constants are so adjusted that within the limits of the operating range of the amplifier the ratio of the direct-current component to the alternating-current component remains unchanged, thus sustaining the light modulation of the lamp at 100 percent.

The photoelectrically controlled neon lamp thus supplies a pulsating beam of light B_4 whose amplitude is proportional to the color sensitivity

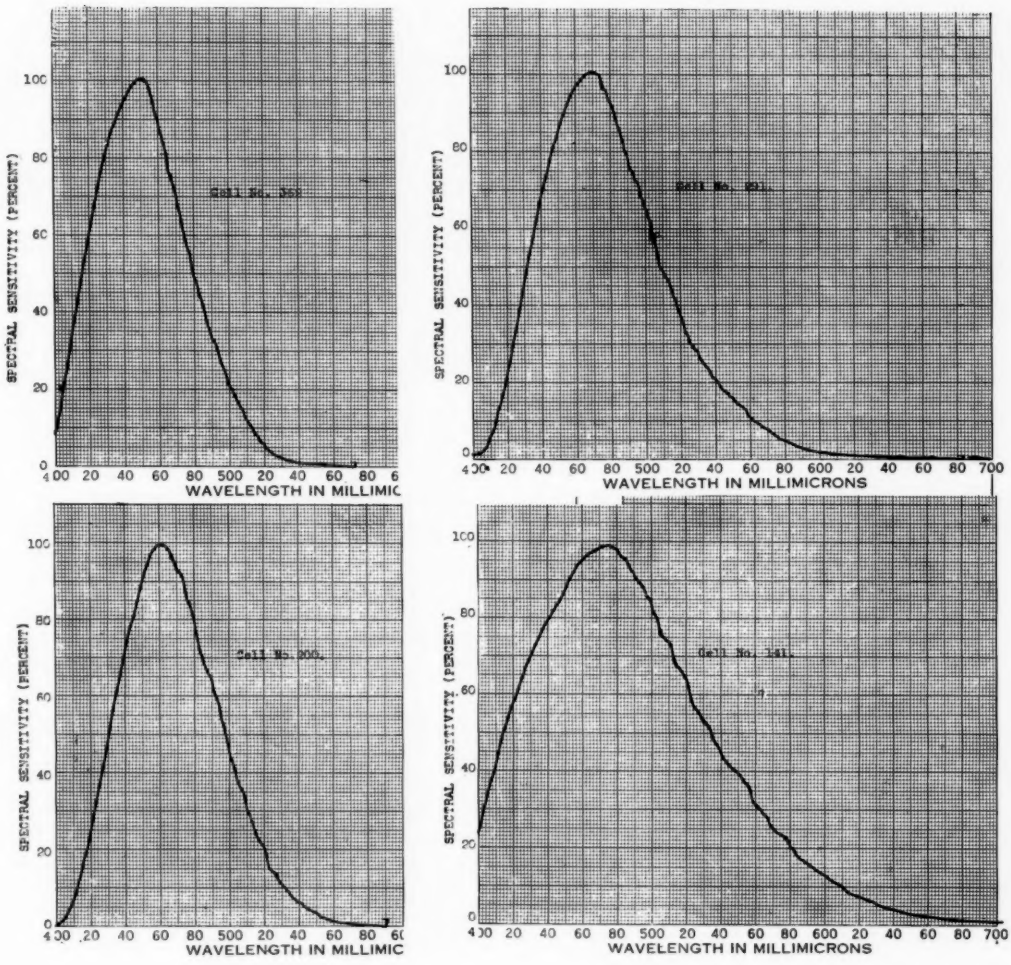

FIG. 3.

of the investigated photo-tube N, and which enters the integrating sphere and acts upon the detector photo-tube.

The a.c. power supply to the reference light source G which produces the pulsating beam B_2 and the a.c. supply to the synchronous motor driving the Rochon prism No. 2 which produces the pulsations of the beam B_4 are two 60-cycle sources 90° out of phase with each other. Consequently the beams B_2 and B_4 also pulsate 90° out of phase with each other.

Thus all the conditions established for intensity variations of the two beams B_1 and B_3 coming from the monochromator of the Hardy spectrophotometer, have been complied with in controlling the beams, B_2 and B_4.

Consequently the same physical relations required for the proper operation of the thyratrons controlling the recording pen are reestablished by the inserted adaptor.

A general view of the complete apparatus including both the spectrophotometer and adaptor for recording spectral sensitivity of photoelectric surfaces is given in Fig. 2. The component parts

of the adaptor are marked with letters corresponding to those given in Fig. 1.*

Operating Conditions

For the recording of color sensitivity the following steps in the operation of the apparatus must be taken.

1. Phase relation. By choosing the proper polarities of the Variac L and for the synchronous motor driving the Rochon prism No. 2 it is possible to check the direction of the motion of the recording pen along the rotating drum. By interrupting either the beam B_2 or B_4, the proper polarities at the Variac can be determined. These polarities must be such that the interruption of the beam B_2 causes the pen to travel toward the higher values of the relative sensitivity ordinate, while interruption of B_4 makes the pen move toward the lower limit of the ordinate.

2. Beam intensities. The intensities of the beams must be so chosen that they stay within the limits of the sensitivity of the detector photo-tube without producing charges large enough to cause blocking of the thyratron grids. By means of a proper reflecting surface at U and a variable aperture V at the neon source Q, the absolute value of the beam intensities, as well as their ratios, should be adjusted for the least sluggishness of the recording mechanism.

3. Amplifier adjustment. Before the recording is started, the wave-length λ_M for maximum sensitivity must first be established. This is done by observation of the photometer scale indicator which is coupled with the pen. Having found this wave-length, the amplifier gain is adjusted so that the ordinate reading on the coordinate paper is 100 percent for the maximum wave-length λ_M. This is the characteristic point at which the intensities of the two beams B_2 and B_4, are equal at the detector photo-tube.

4. Recording. The color sensitivity curve is then taken by setting in motion the recording mechanism which allows the wave-length to vary from 400 to 700 millimicrons while the motion of the pen is controlled by the periodic variations of the photoelectric current supplied by the detector photo-tube.

* All parts of the adaptor are readily removable when transmission or reflectance curves are to be taken.

Examples of Recorded Spectral Sensitivity Curves

Recordings of four typical photo-tubes have been chosen to show spectral sensitivity in relation to the treatment of the sensitized surfaces. These curves are given in Fig. 3. The photo-sensitive surfaces of all four cells consisted of a deposited layer of potassium. The process of sensitization, however, differed in each case. Cell No. 362 was sensitized by means of atomic hydrogen obtained by thermal dissociation of molecular hydrogen. Cells Nos. 200 and 291 were sensitized by means of electrical discharges in water vapor,[3] but the procedures for each of the two tubes differed. Finally, Cell No. 141 was sensitized with electrical discharges in deuterium oxide vapor (heavy water).[4]

The curves indicate clearly the shift of the λ_M (wave-length for maximum sensitivity) for each of the sensitized potassium surfaces.

They also show how the different treatments of the potassium surfaces affect the spectral sensitivity distribution within the visible spectrum.

The Hardy instrument corrects automatically for equal dispersion but is not provided with a device for reducing the intensity of the beam from the tungsten source to equal energy at each wave-length band of constant width. Consequently, the above curves in Fig. 3 need correction by a variable factor depending upon the spectral energy distribution of the source. Although this correction is not considerable due to the high color temperature at which the lamp is operated (C.T. = 3500°K approximately), an improvement in this respect will be obtained by providing means for constant energy distribution as planned.

A detailed report of the apparatus and its application to various types of photoelectric surfaces including barrier photo-cells will be published in a bulletin of the University of Illinois Engineering Experiment Station.

The authors wish to acknowledge the helpful cooperation of Professor J. O. Kraehenbuehl in the operation of the spectrophotometer and express their appreciation to him for the use of the facilities of the Illumination Laboratory.

[3] J. T. Tykociner, Jakob Kunz, and L. P. Garner, University of Illinois Eng. Exp. Sta. Bulletin No. 325 (1940).
[4] J. T. Tykociner and L. R. Bloom, Phys. Rev. **57**, 571 (1940).

Spectrographic Analysis of Briquetted Tablets*

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INTRODUCTION

MUCH of the material submitted to a laboratory for analysis is in the form of a powder, grains, or chips. In these forms a sample of material to be analyzed by a spectrograph does not lend itself well to excitation by high voltage spark or a.c. arc. When a sample in these forms is pressed into a firm and solid tablet, a uniform surface is presented for electrical excitation, under which conditions accurate analysis may be made with a spectrograph. The purpose of this paper is to describe suitable equipment to form briquetted tablets and describe spectrographic test procedures.

PREPARATION OF SAMPLE

It is desirable to obtain the sample in as finely ground condition as practicable. Materials that are not conductors of electricity should be ground to pass a 150-mesh screen and then thoroughly mixed with a definite percentage of a good conductor of electricity such as powdered sucrose, carbon, or metals such as iron or copper oxide.

Some materials require the addition of a binder to form a firm tablet. A binder that works well is sugar in quantities of about 10 percent.

When the quantitative analysis is based on an internal standard, and the sample does not contain a suitable element that may be used as an internal standard, it is practicable to add an element such as powdered iron. An addition of 80 percent powdered iron has proven very successful, as will be shown in succeeding paragraphs.

In the case of soft metals such as non-ferrous white metals and gray iron, no particular sizing of samples is required, except that chips submitted for samples should fall between $\frac{1}{32}''$ and $\frac{3}{32}''$ in length, thus facilitating filling the mold and permitting a large number of particles to form the face of the tablet being subjected to electrical excitation.

BRIQUETTING MACHINE

A very important step in using the powdered, granular, or chip forms of sample for spectrographic analysis is to have available a suitable briquetting machine. The diameter of the specimen should preferably be small, and it should be briquetted at a high pressure. The load applied to the mold and plunger must be applied axially to avoid bending stresses which would cause failure of the plunger.

The briquetting machine illustrated in Fig. 1 is built especially for forming briquetted tablets $\frac{1}{4}''$ or $\frac{1}{2}''$ in diameter and $\frac{1}{4}''$ in length.

A motor-driven hydraulic loading mechanism is employed to load the piston of the press. The oil-loaded piston forces the plunger up into the mold. The sample to be briquetted is placed in the mold, the top of which is funnel-shaped to allow easy filling of the mold. The top of the mold is closed by a short plug. The top plug is held in position by means of a pivoted stop. Moving the front control lever to the left causes the piston to be loaded with oil pressure and the mold plunger is then loaded with a pressure selected by the operator. The pressure applied to the plunger is automatically controlled by a valve, or it may be regulated manually. A pressure of 12,000 lb./sq. in. on a $\frac{1}{4}''$ diameter specimen of ferrous metal powder has proved satisfactory, while a pressure of 7000 is sufficient for non-ferrous metals. The pressure applied is indicated by a gauge. A one-gram sample of ferrous metal will form a $\frac{1}{4}''$ long briquetted tablet.

METHOD OF HOLDING BRIQUETTED TABLET IN GAP

Two briquetted tablets are ordinarily used, one for the upper electrode and another for the lower electrode. Since the tablets are only $\frac{1}{4}''$ in

* Presented at the Ninth Summer Conference on Spectroscopy and Its Applications at Massachusetts Institute of Technology, Cambridge, Massachusetts, July 21–23, 1941.

length, some form of tablet holder is necessary. In Fig. 2 are illustrated tablet holders that have proved very successful. They may be made of brass, copper or stainless steel. The large head which grips the tablet is $1\frac{1}{2}''$ in diameter and $\frac{5}{8}''$ in height. This head is mounted on a $\frac{1}{4}''$ diameter stem on which the electrode holder of the arc-spark stand of the spectrograph is clipped.

The tablet holder is split into two parts. The lower portion of the stem is left intact. This construction gives a spring action to the enlarged portion of the holder so that it will grip the tablet. The tablets are removed from the holder by pushing the holder over the upright stem shown in Fig. 2. The holders are hollow so that the tablet is pushed out by the stem.

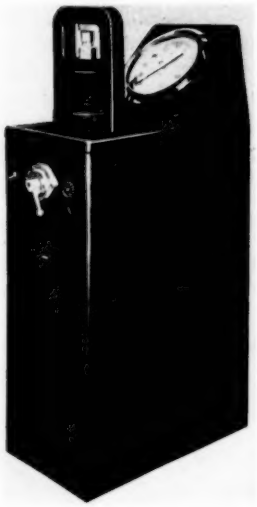

FIG. 1. Briquetting press for forming powders, grains, chips, borings and turnings into firm tablets.

It is permissible to grind ferrous metal or file non-ferrous briquetted samples to a selected contour on the surface to be excited. Tablets of steel or iron grind as easily as cast steel or iron specimens. They may be ground or filed repeatedly for check analysis.

The large mass of metal of the tablet holders keeps the tablets cool, which is very essential for accurate test results. Cooling briquetted samples in this manner produces a greater degree of accuracy than cooling the tablets by an air blast. An air blast on a large diameter specimen, for example $\frac{1}{4}''$, blows the spark around considerably. A spark on a small diameter specimen is held in place by the small surface, thus enabling one to cool it with an air blast.

TEST RESULTS

A series of test data will be presented to illustrate some of the requirements in excitation that are necessary to obtain a high degree of accuracy in the analysis of metals and slags. An ARL-Dietert grating spectrograph was used for all of the tests.

In a drop forge plant, the steel is received in the form of bars. A good representative sample is obtained from a bar of steel by milling one surface for a distance of one foot. The milled chips are kept short by using a milling cutter having a large number of chip breaker grooves on its cutting edges. The milled chips were briquetted into tablets $\frac{1}{4}''$ in diameter and $\frac{1}{4}''$ in length. The tablets were ground on one end to a cone with a 5° angle. Pronged electrode holders were used to grip the specimen during the grinding and excitation.

The results as tabulated on Table I were obtained by using an air blast. A comparison between the spectro- and chemical analysis shows that closer agreement is desirable. Methods of improving accuracy will be discussed in succeeding paragraphs.

Many standard samples are available from the National Bureau of Standards. To illustrate the adaptability of these standard samples for spectro-analysis by the briquetting method, the material presented in Table II is of interest. The specimens were held in pronged holders and

TABLE I. *Comparative analysis, spectro- and chemical analysis of $\frac{1}{4}''$ briquetted drop forge steel specimens.*

SAMPLE NUMBER	% CONCENTRATION 7139	7234	7262	7354
	Silicon			
Chemical analysis	0.23	2.03	0.23	0.26
Spectrographic	0.203	2.07	0.286	0.27
	Manganese			
Chemical analysis	0.79	0.77	0.81	0.61
Spectrographic	0.74	0.80	0.78	0.60
	Chromium			
Chemical analysis	0.03	0	0.84	0.07
Spectrographic	0.22	0	0.83	0.09
	Nickel			
Chemical analysis	0.055	0	1.21	1.88
Spectrographic	0.060	0	1.20	1.90

air cooled. This method of cooling specimens does not yield the best results as will be shown in succeeding paragraphs. A 300-mesh screen was placed in the light path ahead of the slit, which produces much better results than a lens.

The data tabulated in Table III illustrate that the accuracy of spectro-analysis of briquetted specimens may be readily improved.

A reduction in exposure time from 50 seconds to 20 seconds gives a material improvement in accuracy. Thus, short exposures are recommended for briquetted samples. Inaccuracies associated with long exposures appear to be due to the heating of the specimens, causing diffractional distillation of the various elements.

On a long exposure, an air blast does not cool the surface sufficiently to prevent formation of the incandescent material on the surface of the specimen.

TABLE II. *Spectro-analysis of a series of National Bureau of Standards samples.*

Bureau of Standards Sample No.	Analysis Bureau of Std.	Analysis Spectrograph	Percentage Difference
	Analysis of silicon		
4c	1.26%	1.27%	0.8%
4e	1.29	1.25	3.1
5c	1.85	1.85	0.0
7c	1.79	1.79	0.0
122	0.54	0.54	0.0
5g	1.84	1.89	2.7
	Analysis for copper		
4c	0.234%	0.235%	0.0%
4e	0.010	—	—
5c	0.093	0.09	0.0
5g	1.44	1.46	1.4
5g	1.44	1.43	0.7
	Analysis for manganese		
4c	0.897%	0.84%	6.4%
4e	.721	.72	0.0
5c	.806	.81	0.5
5g	.610	.63	3.3
7c	.564	.56	0.7
122	.510	.48	5.9
5g	.610	.61	0.0

A short spark gap between the specimens does not reduce the accuracy, for example—reducing the gap from $\frac{1}{8}''$ to $\frac{1}{16}''$ is beneficial as shown in Table III.

The data of Table III show that a condensing lens on the optical bench is not required for the $\frac{1}{4}''$ diameter tablets. For example, average error with lens in light path is 2.5. Under identical conditions, with lens out of the light path, the average error is reduced to 1.55 percent.

FIG. 2. A briquetted steel tablet is shown held in the right hand and a massive tablet holder is shown held in the left hand.

The accuracy is further improved when the briquetted specimens are cooled, not by air which blows the spark around, but by the heat absorption of the massive specimen holders.

A high degree of spectro-analysis accuracy may be obtained by using a briquetted steel when the sample gap is 1/18", 60-micron slit, no lens, without an air blast, with 300-mesh screen and specimen cooled by massive specimen holders. As an example of the accuracy that may be obtained over a period of time, the data tabulated on Table IV are illustrative. The data show that the analyses are reproducible.

Many materials may be formed into tablets for spectro-analysis. As an example, one may choose slag which is generally considered as a complex material and difficult to analyze. Since the briquetter allows one to use a powdered sample, it is practicable to add a definite quantity of some element which may be used as in internal standard and also as a binder and conductor of electricity. In the case of the cobalt slag under discussion, 75 percent iron powder was added which served both as a binder, conductor for the electric current and as the internal standard. Briquettes were formed by grinding

TABLE III. *Exposure time, air blast and electrode separation influence reproducibility.*

Spectrogram No.	Exposure Time	Spark Gap	Air In. Hg	Lens	Screen	Average Error
1	80 sec.	$\frac{1}{8}''$	3	Out	In	6.0%
2	40 sec.	$\frac{1}{8}''$	3	Out	In	5.2%
3	20 sec.	$\frac{1}{16}''$	3	Out	In	2.0%
4	10 sec.	$\frac{1}{16}''$	0	In	Out	2.5%
5	10 sec.	$\frac{1}{16}''$	0	Out	Out	1.55%

TABLE IV. *Spectro-analysis accuracy attainable over a period of several days on briquetted steel drillings.*

Sample No.	Silicon Wet Chemical Analysis	Spectrographic Analysis Film 410	Film 411	Film 412	Film 415
3639	1.99%	2.05	1.95		1.95
3645	.97				2.02
9235	.21	.21	.20		.21
9238	.25	.26	.25	0.24	.26
9240	.24	.23			.23
9248	.24	.24	.24		.23
9249	.22	.22	.24		.19
9500	.18	.17			.18
9628	.08				.09

0.250 gram of cobalt slag with 0.750 gram of C.P. powdered iron in an agate mortar and briquetting the mixture into $\frac{1}{4}''$ diameter specimens in an hydraulic press at 15,000 lb. Two briquettes were made; one to serve as the upper and the other the lower electrode. The specimens were ground to an obtuse conical point.

CONCLUSIONS

The field of spectro-analysis is considerably broadened, without sacrificing accuracy, by employing the briquetting of specimens as described in this paper. The technique required is not far removed from that used by cast or machined samples or for those samples placed in solutions.

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A Physical Photometer

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Photometry has been carried on for many years by means of visual observations not because the eye is an accurate measuring instrument but because there has been no physical apparatus which would respond with sufficient accuracy to light in the same manner as the human eye. Even experienced observers may obtain markedly different results because they will not agree as to when two illuminated surfaces are equally bright. A physical photometer which "sees" light in accordance with the ICI luminosity factors has been constructed. These luminosity factors have been accepted internationally to represent the characteristics of the eye of a "standard observer." This photometer gives results which are consistent with the ICI luminosity factors and which are more accurate than those of visual observers when the light transmissions of colored filters are measured. The construction and operation of this physical photometer are described in a paper by R. P. Teele in the National Bureau of Standards Journal of Research, **127**, No. 3 (RP 1415).

I. INTRODUCTION

PHOTOMETRY has been carried on for many years by means of visual observations, not because the eye is an accurate measuring instrument but because there has been no physical apparatus which would respond to radiant energy in the same manner as the human eye with an accuracy comparable to that of visual observations made under the best conditions.

Vision is a combined physiological and psychological process. In the visual photometry of colored lights, otherwise normal observers may obtain markedly different positions of photometric balance because of their different brightness evaluation of the different parts of the spectrum.

Experience also shows not only that the results obtained in heterochromatic photometry are influenced by the characteristics of the eye of the observer, but that nearly all observers using the equality-of-brightness method will change their decision as to a photometric balance over a period of a month or more. Some observers even exhibit a variation in this decision during a single day.

Even in homochromatic photometry the results obtained are influenced by the attitude of the observer (for example, lack of concentration caused by worry or day dreaming), and by his ocular and bodily fatigue or discomfort, and the immediate prior use of an observer's eye must be controlled before observations are made; for example, an observer entering a photometric laboratory from a brightly lighted room requires a period of time to become properly adapted to the light conditions of the photometer room. All of these variations make it desirable to devise a physical instrument to evaluate light.

Various experimenters have worked on the problem. It is sufficient here to note the experiments of Ives and Kingsbury, and of Ives,[1] in 1915, in which a luminosity filter and a thermopile were employed. The luminosity "filter" in one case was a template which was placed in the path of the radiant energy dispersed by a prism. The optical system required and the mechanical difficulties encountered in constructing, mounting and aligning a template have made this method impractical. In the other case the luminosity filter consisted of a liquid solution. In both cases a Thomson galvanometer was used. The susceptibility of this galvanometer to mechanical disturbances and drift of zero, and the lack of exact proportionality between current and deflection in the circuit, imposed serious limitations on their measurements. In order to avoid the difficulties which Ives and Kingsbury encountered, which are present even with the best of modern galvanometers, it was decided in the present work to measure the e.m.f. developed, instead of drawing and measuring current from the thermopile. Since this e.m.f. is small it was necessary to develop a special potentiometer and procedure in order to make the measurements to the required accuracy, and these have been described in a previous paper.[2] The present paper gives a more complete description of the physical photometer which was constructed, and describes the method of operation and the results of tests which were made to determine its performance.

[1] H. E. Ives and E. F. Kingsbury, Phys. Rev. **6**, 319 (1915); H. E. Ives, Phys. Rev. **6**, 334 (1915).

[2] R. P. Teele and S. Schuhmann, J. Research Nat. Bur. Stand. **22**, 4 (1939), RP1195.

II. The Physical Photometer

The most fundamental concept in photometry is that of luminous flux. Luminous flux is the rate of passage of radiant energy evaluated by reference to the luminous sensation produced by it. If we have a source emitting equal amounts of radiant energy per unit wave-length throughout the visible spectrum (called an equal-energy spectrum) and have observers evaluate the brightness of this energy at each wave-length, we find that the human eye is not equally responsive to radiant energy throughout the visible spectrum. We find two things: (1) All normal observers find the region of greatest luminosity to be in the yellow-green part of the spectrum, the luminosity gradually decreasing in either direction, but (2) there are important differences in the spectral luminosity curves for different normal observers, which result in their different evaluations of the brightnesses of colored lights. The average spectral luminosity curve of the eye at different wave-lengths has been determined for a large number of observers, and an average set of relative values has been accepted by the International Commission on Illumination as a definition of a "standard" photometric observer.[3] The luminosity curve showing the response of this ideal observer for energy received at different wave-lengths is shown as a solid line in Fig. 1.

No physical detector has been constructed which has the same relative response curve as the average human eye for radiant energy of different wave-lengths. It is possible, however, to make a non-selective receiver, which indicates the amount of received energy per unit time independent of its wave-length. In order to have this non-selective receiver give the same luminous response as the eye of the "standard" observer, it is necessary to interpose between the source and receiver a filter which has at each wave-length a transmission proportional to the luminosity factor for that wave-length. Such a luminosity filter, together with a non-selective receiver and an indicator or measuring device

[3] A résumé of the data on which the standard ICI luminosity factors are based and of the present status of these factors is given in a paper by Kasson S. Gibson, "Spectral luminosity factors," J. Opt. Soc. Am. **30**, 51 (1940).

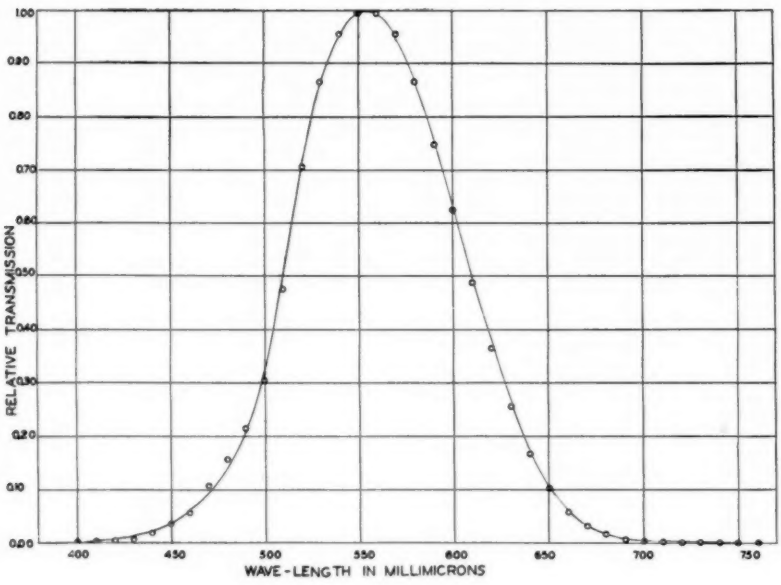

FIG. 1. The relative spectral transmission for the luminosity filter (circles) compared with the standard ICI luminosity curve (continuous line).

attached to the receiver, constitutes a physical photometer which will measure the rate of passage of radiant energy evaluated by reference to the luminous sensation produced by it, i.e., will "see" radiant energy in the same manner as the eye of the standard ICI observer.

a. Luminosity filter

The luminosity filter first realized[4] for use as part of the physical photometer was found to lack satisfactory reproducibility, and a second filter,[5] which tests indicate is sufficiently reproducible, was devised. The extent to which the relative transmission of the second luminosity filter duplicates the accepted ICI luminosity curve in the wave-length interval from 380 to 770 mμ is shown in Fig. 1. The solid line is the ICI luminosity curve and the plotted points are the transmissions obtained from spectrophotometric data for the luminosity filter. A slight amount of radiant energy is also transmitted by this filter in the infra-red region from 800 to 1320 mμ. The method of correcting for this unwanted band of transmitted energy is discussed in Section III.

The filter consists of a piece of glass used in conjunction with a liquid solution. The glass used is a Corning uranium (canary) glass of such a thickness as will give a transmission at 486 mμ of 0.40 ± 0.01. The solution has the following composition:

Copper sulphate, $CuSO_4 \cdot 5H_2O$	20.00 grams
Cobalt ammonium sulphate, $CoSO_4(NH_4)_2SO_4 \cdot 6H_2O$	2.05 grams
Potassium dichromate, $K_2Cr_2O_7$	0.150 gram
Sulphuric acid, H_2SO_4 (1.835 sp.gr.)	10 ml
Distilled water to make 1 liter of solution.	

The solution must be used in a cell of 5.00 centimeters length, which should be placed between the illuminant and the uranium glass (to reduce fluorescence of the latter). Further details regarding the luminosity filter will be published separately.

b. Receiver

The choice of receiver is dependent upon the reproducibility of its indication, the selectivity, the sensitivity, and the linearity of response with respect to the rate of reception of radiant energy. There are a number of types of receivers

[4] K. S. Gibson, R. P. Teele, and H. J. Keegan, J. Opt. Soc. Am. **28**, 178 (1938).
[5] K. S. Gibson, R. P. Teele, and H. J. Keegan, "An improved luminosity filter," J. Opt. Soc. Am. **29**, 144 (1939).

whose response to radiant energy is reproducible and this requirement does not limit to any considerable extent the choice of receiver. Their selectivity to radiant energy rules out from consideration in the present case receivers such as photo-tubes and barrier-layer photo-cells. Although it might be possible to find a combination of a particular selective receiver and filter which together would respond to radiant energy in agreement with the luminosity factors, such a solution to the problem is not yet generally satisfactory because selective receivers even of the same type have differing spectral characteristics. A combination of a filter and a barrier-layer cell which approximates the desired results has been worked out by one manufacturer. This combination is very useful for many purposes which do not require a high accuracy and refinement in its use have recently been described.[6] In the case of non-selective receivers such as thermopiles, bolometers and certain radiometers, the sensitivity and the range over which the response is linear need to be considered together. However, the most sensitive radiometers are either susceptible to various disturbing influences or their range of linearity is too limited. At present there seems to be little choice between bolometers and thermopiles. A thermopile was chosen as the receiver for the present work.

The thermopile used is of the Moll surface type and consists of 80 thermojunctions of constantan-manganin arranged in a circle of 2-centimeters diameter. The thermopile is designed for use in air and its resistance is approximately 60 ohms.

c. Potentiometer

The response of the thermopile when used in the physical photometer was found in preliminary experiments to be of the order of magnitude of 10 microvolts and the special potentiometer was accordingly designed to measure such electromotive forces. In work with the completed physical photometer it was found that measurements covering a range from a few tenths of a microvolt to over 100 microvolts would be necessary in practical photometric work. The potentiometer is capable of covering this range without modification of the original design.

In the measurement of small electromotive forces the most serious difficulty lies in the presence of extraneous thermal electromotive forces in the various parts of the measurement circuit. The major part of the apparatus, comprising the potentiometer, was designed to prevent or eliminate the effects of such electromotive forces.

The galvanometer was selected to work with the combination of thermopile and potentiometer. The potentiometer circuit is such that its resistance is of secondary importance and a galvanometer was specially built to be critically damped by the thermopile, that is, the critical damping resistance of the galvanometer is approximately equal to the resistance of the thermopile. The galvanometer has, of course, a circuit which is all of copper. The sensitivity is 12 mm per microvolt for a scale distance of 1 meter. The galvanometer is used with a scale distance of 10 meters and the circuit is always reversed to obtain an indication of balance of the potentiometer, which in effect doubles the sensitivity. These two expedients give an effective sensitivity of 240 mm per microvolt in actual use, and a change of 0.24 millimeter upon reversal corresponds to 0.001 microvolt.

d. Precautions to be observed

Obviously it is desirable to locate the galvanometer on a vibrationless support. An "*A*" frame support fastened to a heavy masonry wall was found to be adequate for a reasonable percentage of the time.

The thermopile-galvanometer circuit is subject to such disturbances as would be caused by neighboring electrical devices, electrostatic and electromagnetic influences, etc. The thermopile should be electrostatically shielded and located as far as possible from all electrical equipment. The electrostatic shielding must include the galvanometer circuit. The wires of the thermopile-galvanometer circuit must be fastened down at closely spaced intervals to prevent swinging or movement that would change the magnetic

[6] L. E. Barbrow, "A photometric procedure using barrier-layer photocells," Nat. Bur. Stand. J. Research 25, 703 (1940); RP 1348.

field included by them. Festoons or loops of wire are to be avoided and the use of twisted pairs of wires is desirable.

III. Method of Taking Observations

As already stated, the filter transmits radiant energy slightly in the region from 800 to 1320 mμ in addition to its principal transmission in the visible region. This unwanted energy will also be measured by the receiver. The entire response of the receiver is, therefore, proportional to the radiant energy in the visible region plus that in the infra-red. By inserting a filter which does not transmit any visible energy but has a high transmission in the infra-red region, the band of energy in the infra-red may be evaluated. An infra-red (Corning No. 255, sextant red glass) filter will separate the two regions adequately. The measurement must be made in two parts: (1) a measurement of the total energy transmitted by the luminosity filter, and (2) a measurement of the energy transmitted by the luminosity filter and the infra-red filter used together.

If we designate the response of the thermopile for the respective measurements outlined above by P_{LT} and P_{LI} we have

$$P_{LT}=k(E_{LV}+E_{LI}), \tag{1}$$

$$P_{LI}=kE_{LI}F_I, \tag{2}$$

in which k is a proportionality constant, E_{LV} is the energy from the source transmitted by the luminosity filter in the visible region only, E_{LI} is the energy from the source transmitted by the luminosity filter in the infra-red region only, and F_I is the infra-red transmission factor for the infra-red filter. From (1) and (2) it is seen that the response, P_{LV}, in the visible region alone is

$$\begin{aligned} P_{LV}=kE_{LV}&=P_{LT}-kE_{LI}\\ &=P_{LT}-P_{LI}/F_I. \end{aligned} \tag{3}$$

As a preliminary to the use of the physical photometer, it is therefore necessary to determine the infra-red transmission F_I of Eqs. (2) and (3) of the infra-red filter to be used. This may be done by using the physical photometer to measure the transmission of the infra-red filter for energy from an incandescent source passing through a similar infra-red filter. This determination has been carried out for incandescent sources over the color-temperature range 2046 to 2842°K and F_I was found to be constant at 0.70 within ±0.01. For such sources this uncertainty in F_I introduces at most an error of 0.2 percent of the luminous flux. Therefore the determination of F_I for the usual incandescent source can be made at an intermediate color temperature once for all.

The total response of the thermopile-galvanometer circuit is caused both by the radiant flux from the source being measured and by stray or extraneous flux from the surroundings. By the surroundings are meant the laboratory room and its contents, including the persons who are operating the equipment.

The electromotive force caused by the radiant flux from the source is distinguished from the rest by the use of a shutter to allow the flux being measured to reach the receiver at desired times. The shutter used should be designed and placed so that its operation will not change the conditions of the surroundings or cause air currents around the thermopile. In the present work a rotary shutter made from a disk of plywood with an opening to let the energy pass through to the receiver proved to be satisfactory. By placing this shutter between the source and the luminosity filter, the latter can be used as a window for the thermopile housing and thus reduce materially the trouble from air currents which would be encountered in using an uncovered thermopile.

The surroundings are, in general, not at the same temperature as the thermopile and thus there is an exchange of radiant energy between the thermopile and the surroundings. The thermoelectromotive force of each of the junctions is about 40 microvolts per degree centigrade and hence the total for the 80 junctions is more than 3000 microvolts per degree centigrade. Since it is sometimes desired to make measurements imprecise by less than 0.001 microvolt the receiver surface and the dark junction cannot be considered to be at the same temperature unless their temperatures differ by less than about 3×10^{-7} of a degree centigrade. It is not practical to maintain these temperatures within such small limits and it is, therefore, necessary

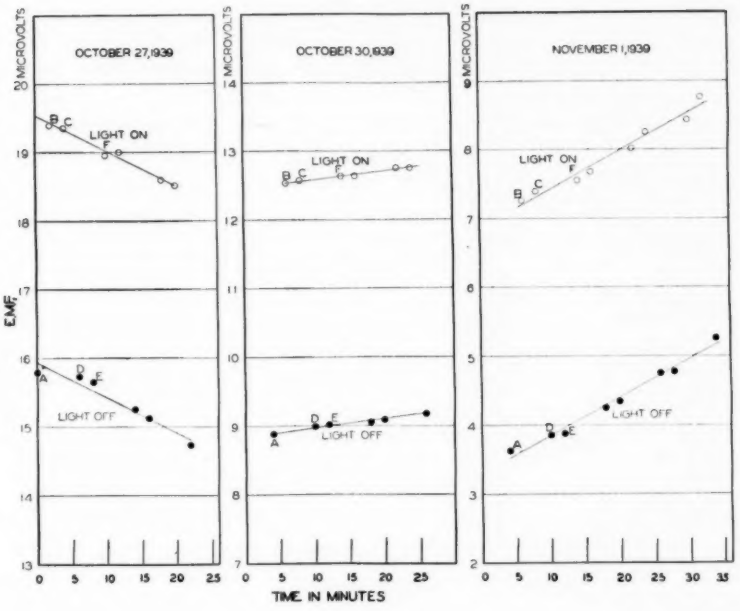

FIG. 2. Response for a 300-watt lamp at a distance of 1.56 meters at constant voltage. In this case observations were taken at 2-minute intervals.

to eliminate the effect of the stray energy by an accurate timing of the measurements of the response when the receiver is screened from and when it is open to the energy from the source. Gradual warming or cooling of the surroundings causes a first order drift in response due to stray energy, and second order changes are avoided by taking precautions to prevent sudden changes in conditions of the surroundings, i.e., by keeping doors and windows closed, by having the operators located at a distance from the thermopile housing and by keeping equipment located in one position during a series of measurements.

Figure 2 shows readings taken with a lamp at a fixed distance on different occasions. These readings are taken in the following manner. The light source is turned on and the potentiometer approximately balanced with the shutter closed. Temperature equilibrium is reached in the course of 2 or 3 minutes. An accurate balance is then obtained and the shutter opened. After a definite time interval, say 2 minutes, another reading is taken. The shutter is kept open for the chosen time interval and another reading taken and the shutter closed. After 2 minutes a reading is taken. The shutter is kept closed for an additional 2 minutes and a reading taken. This cycle is continued for a group of three or four pairs of readings. The time interval will depend to some extent upon the ability of the potentiometer operator but should be as short as is consistent with the determination of accurate balances of the thermopile-galvanometer circuit and of the potentiometer. The time necessary for the thermopile to reach thermal equilibrium determines the shortest permissible interval. For the thermopile used this time is approximately 10 seconds. A time interval of 1 or 2 minutes was found to be satisfactory.

Figure 2 is for illustrative purposes only and in actual use a definite time interval is used and the average of two adjacent light-off readings is subtracted from the average of the corresponding adjacent pair of light-on readings to obtain the response to the energy being measured; for example, the average of points *A* and *D* subtracted from the average of points *B* and *C*. Obviously, the average of points *D* and *E* subtracted from the average of points *C* and *F* also represents the response to the energy being measured. The three days shown in the figure include a day on which the initial light-off

reading was comparatively large and the drift was negative, a day when the drift was small and a day when the drift was strongly positive.

The data shown in Fig. 2 indicate that results of fair precision may be obtained from a light-off, light-on and light-off reading at approximately equal time intervals. It is also obvious that when the response is large, say 50 to 100 microvolts, the necessity for accurate timing is greatly reduced. For example, in the section of Fig. 2 for November 1, 1939, the entire time is over half an hour and it is apparent that if a light-off reading were taken at point A, and a light-on and light-off reading were taken at any time during the next half hour, an error of less than 1 microvolt would be caused by neglecting time altogether and considering the average of the two light-off readings as representing the light-off conditions. If the vertical separation were 100 microvolts, as it may often be, this 1 microvolt would represent an error of only 1 percent. Usually readings may be taken in a time interval of a few minutes rather than half an hour. If an uncertainty of 1 or 2 percent may be tolerated it is possible to simplify the procedure and use more rugged, but less sensitive, electrical equipment.

IV. Experimental Tests

The physical photometer was tested experimentally by measuring spectrally selective filters to verify the adequacy of the luminosity filter and thermopile and by measuring rotating sector disks to be sure that rotating lamps would be evaluated correctly.

a. Measurement of spectrally selective filters

The glasses measured were chosen from types that are very selective in their transmission and were so selected as to include transmission bands distributed throughout the visible spectrum. The group included those designated as selenium red, selenium yellow, signal yellow, cobalt blue, signal green and signal blue, the glasses ranging from 2 to 50 percent in transmission, and also signal-purple filters some of which transmitted only one-half of 1 percent of the incident light from an incandescent lamp at 2848°K. Their spectral transmissions and the computed luminous transmissions had been carefully determined by others and the data for many of them have been published.[7]

In the measurement of the luminous transmission factor F_V of a glass filter, four measurements have to be taken, corresponding to Eq. (4) with the glass in the beam and with it removed. The value of F_V is given by the ratio

$$F_V=\frac{(P_{LT}-P_{LI}/F_I)_{\text{glass in}}}{(P_{LT}-P_{LI}/F_I)_{\text{glass out}}}. \qquad (4)$$

The comparison of the results of measurements made with the physical photometer according to Eq. (4) and the values obtained by computation from spectrophotometric data by the use of the ICI luminosity factors is given in Table I. The values calculated from spectrophotometric data were computed by the usual summation method with 10-millimicron intervals, although this computation by discrete intervals may slightly reduce the accuracy of the final result. The response of the physical photometer is equivalent to a continuous integration rather than to the summation used in the computations.

Table I. *Comparison of values of luminous transmission of spectrally selective glasses obtained by direct measurement with the physical photometer and by computation from spectrophotometric data with ICI luminosity factors.*

Color Designation of Glass	Luminous Transmission for 2848°K	
	Physical Photometer	Spectrophotometer
Selenium red	0.088	0.089
Selenium red	.103	.101
Selenium red	.135	.139
Seleium yellow	.505	.500
Signal yellow	.378	.376
Signal yellow	.506	.508
Cobalt blue	.163	.161
Cobalt blue	.244	.242
Cobalt blue	.291	.290
Cobalt blue	.374	.374
Cobalt blue	.402	.400
Signal green	.182	.184
Signal green	.200	.201
Signal green	.251	.252
Signal green	.268	.270
Signal blue	.0356	.0356
Signal blue	.0215	.0216
Signal purple	.0049	.0051
Signal purple	.0050	.0049
Signal purple	.0063	.0059
Signal purple	.0115	.0120
Signal purple	.0200	.0206

[7] K. S. Gibson and Geraldine K. Walker, Signal Section, Proceedings A.R.A. **30**, 405, 420 (1933).

On the other hand, the physical photometer data were obtained at room temperatures whereas the spectrophotometric data, on which the computations are based, were made at 25°C. It so happens that the selenium red and yellow glasses which are subject to appreciable summation errors, are also the least uniform in transmission (through different parts of a given filter) and subject to the largest temperature effects.[8] The discrepancies between the two columns of values are well within the combined experimental errors of the two methods for all of the glasses.

TABLE II. *Comparison of transmissions of sectored disks.*

Disk Designation	Transmission Factor Calculated from Angular Opening	Transmission Factor Measured by the Physical Photometer
10 per cent	0.1000	0.1001
1 percent	.00979	.00975

The test with these 22 filters justifies the conclusion that the physical photometer gives results which are consistent with the ICI luminosity factors and verifies the performance of the combination of luminosity filter and thermopile-galvanometer-potentiometer system.

b. Measurement of rotating sectored disks

When sectored disks are used the light reaching the receiver varies from zero, when the opaque portion is in the path of the light, to full intensity, when the open portion is in the path of the light. Obviously, the use of sectored disks is a more severe test of the physical photometer than the evaluation of a rotating lamp for which the changes in intensity are small.

The transmissions of two sectored disks, one with four openings, transmitting about 10 percent, and one with two openings, transmitting about 1 percent, were measured on the physical photometer. The transmissions of the sectored disks were also calculated from the angular openings.[9] The calculated and measured transmissions are compared in Table II.

[8] The purple glasses are also subject to appreciable summation errors and important temperature effects but are uniform in transmission.

[9] The angular openings were determined by the Length Section of the Weights and Measures Division of the Bureau.

The results given in Table II indicate that the physical photometer will correctly evaluate an intermittent light when the periods of fluctuation are short compared to the response period of the thermopile-galvanometer circuit (about 10 seconds). Any period can be used provided it is not great enough to cause perceptible flicker of the galvanometer spot.

c. Stability

One important consideration is the stability of the physical photometer. In measuring lights it is desirable to have an instrument which can be calibrated and then used for a period of time before recalibration. The length of time between calibrations is governed by the possible aging effects of the component parts of the physical photometer.

The surface of the receiver is well protected from mechanical damage and there is no reason to expect that rapid changes will occur from aging of the surface. The chemicals used in the luminosity filter are all stable in the concentrations used in the solution. Most of the constituents have been carefully studied by other investigators.[10] The properties of the glass are not likely to change.

Nevertheless, the stability was investigated by two separate tests. A seasoned lamp was placed at a fixed distance from the receiver and measured over a period of two weeks. The results are given in Table III. A study of these

TABLE III. *First test of stability.*

Date	Response percent
March 23	100.17
25	99.92
28	100.30
31	100.04
April 1	99.65
6	99.92
Mean	100.00

data shows a maximum variation from the mean of 0.35 percent and an average deviation of 0.17 percent. These variations are within the experi-

[10] R. Davis and K. S. Gibson, "Filters for the reproduction of sunlight and daylight and the determination of color temperature," Misc. Publications, Bureau of Stand. No. 114 (1931).

mental error and this test shows that during this length of time there was no appreciable change in the e.m.f. of the thermopile for a given density of luminous flux.

The second test was made as part of a series of measurements on a group of standardized lamps. The period of time covered by this series of measurements was nearly six months. The response for a given density of luminous flux during this period did not vary by a much as 0.4 percent from the mean and the average deviation from the mean was less than 0.2 percent.

V. Application to Measurement of Lamps

The results obtained in measuring highly selective filters and given in Table I indicate clearly that the physical photometer may be used to measure the light from all spectral types of sources including fluorescent lamps, mercury, neon, sodium and similar gaseous-discharge lamps. Parker[11] has already used a similar physical photometer for work with mercury and sodium lamps.

The values obtained for the rotating sectored disks (Table II) show that fluctuating sources such as rotating lamps and gaseous-discharge lamps (operating on alternating-current circuits) will be correctly integrated when the period of the fluctuation is not long compared to the response period of the thermopile-galvanometer circuit.

The work done with the physical photometer indicates that, when once calibrated, it will enable the candlepower of a lamp to be determined without the use of other lamps as standards and without the use of accessories such as color-matching filters. Visual observers must balance a lamp against other lamps of known candlepower in order to determine the candlepower of the unknown lamp; little time can intervene between the photometric balances for the different lamps; and when the lamps being compared differ in color it is necessary to use accessory filters to equalize the color. The use of a previously calibrated physical photometer to determine the candlepower of a lamp will be of considerable advantage in conserving the life of standard lamps and will result in a saving in the time required to determine the candlepower of a single lamp. In addition, it will be unnecessary to take time to obtain and to calibrate a color-matching filter, when such a filter is required as an accessory for visual observations.

Another advantage of the physical photometer is that one person, experienced in using sensitive electrical measuring equipment, can make the measurements required and thereby save most of the time and trouble involved in obtaining and using a group of visual observers who have photometric experience and nearly normal luminosity functions.

Work leading to the use of the physical photometer for the determination of the candlepower of lamps operating at 2046°K, 2360°K and 2727°K to give three points on a new photometric scale is under way.

VI. Summary

The experimental results show that a physical photometer composed of three elements, a luminosity filter, a spectrally non-selective receiver, and a sensitive electrical measuring apparatus attached to the receiver gives photometric results which are practically the same as those which should be obtained by a standard observer having the characteristics represented by the ICI luminosity factors.

The luminous transmission of a spectrally selective filter can be determined with the physical photometer in a much shorter space of time than that required to obtain the spectrophotometric data and perform the necessary calculations. The two methods are comparable in accuracy. It is often unfeasible to make such determinations by direct visual observation.

[11] Allan E. Parker, "Measurement of illumination from gaseous discharge lamps," Illum. Eng. **35**, 883 (1940).

Radiant Energy Measurements with Thermopiles

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(Received August 22, 1941)

A carbon-coated thermopile always gave smaller readings on ultraviolet lamps when the thermopile was evacuated than when it was filled with air or other gases. The transparency of the carbon coating for long wave-length radiations and its opacity to short wave-lengths, requiring the presence of gas molecules to transport part of the ultraviolet energy from the front surface to the thermal junctions explains the observed phenomenon. A thermopile having a coating of platinum black gave the same values whether evacuated or air filled because a thin coating of platinum black is an excellent absorber for all wave-lengths.

IT is customary to indicate the strength of an ultraviolet lamp by the intensity of radiant energy at a certain distance from the lamp or by the total radiant energy flux from the ultraviolet source. Measurements can be made in absolute units such as microwatts of energy per cm² at one meter's distance from the source with a thermopile which is a non-selective receiver, or with a photo-cell responding to a limited wave-length region that has been calibrated against a thermopile. In the data presented herewith there is a marked discrepancy in the values obtained with a certain thermopile that is evacuated compared to the values measured with the same thermopile filled with air or some other gas. The differences are of the order of 30 to 40 percent and are far greater than experimental error. With another thermopile the evacuated and air-filled readings are equal.

Apparatus and Method

Three thermopiles were used and are designated as *A*, *B* and *C*. They were made by The Eppley Laboratory, Inc. Thermopiles *A* and *B* were coated with carbon while thermopile *C* was coated with platinum black. Each thermopile had twelve copper constantan junctions connected in series and was mounted inside a metal case having a thin quartz window in front of the thermopile element and an exhaust tube so that it could be evacuated if desired. The thermopile was connected to a high sensitivity low resistance L & N galvanometer and was calibrated with a Bureau of Standards certified Standard of Radiation—a carbon filament incandescent lamp, the radiant energy of which had been determined in one specific direction. The galvanometer was checked for linearity of response, and over the range of deflections from 1 mm to 10 cm, the galvanometer sensitivity was constant. Galvanometer deflections were between 1 cm and 6 cm for most readings.

The thermopile-galvanometer combination was calibrated before each set of readings against a Standard of Radiation and occasionally checked against all three of our Standard Lamps. Checking again at the end of a series of observations showed that the thermopiles maintained a constant sensitivity over a period of at least several hours.

A number of mercury vapor lamps were measured by the use of filters that isolated the desired spectral regions. A quartz water cell one cm thick was placed directly in front of the thermopile to eliminate infra-red radiations. A polished fused quartz plate and a barium flint glass plate were alternately interposed into the beam of light, the difference in galvanometer readings being a measure of the ultraviolet energy given off by the lamps. The glass filter designated as BaSF4 was 2 mm thick and had a transmission of approximately 50 percent at λ3340, while the quartz plate had a transmission of about 50 percent at λ2000. The thermopile and filters were mounted inside a wood box about 12 inches on a side to reduce air drafts and temperature variations to a minimum. A simple calculation involving the loss of energy in passing through the various filters gave the ultraviolet energy between the short wave cut off of the glass and quartz filters. The transmission

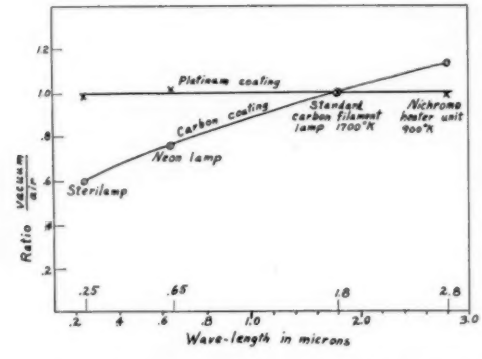

FIG. 1. The response of a carbon-coated thermopile varies with wave-length while a thermopile coated with platinum black has a uniform wave-length response.

of the water cell for λ2537 was 85.5 percent. Freshly distilled water was used each day for if water were allowed to remain in the water cell and be exposed to the air for 24 or 48 hours, the measured transmission for λ2537 radiation would be appreciably less.

The filter method is extremely simple in theory and operation but accurate knowledge must be had of the transmission of the various filters both in the ultraviolet and infra-red regions to give the desired results. Slight errors in transmission factors[1,2] may lead to large total errors and to erroneous conclusions. This method is well adapted to isolate the resonance radiation of mercury at 2537A in certain low pressure discharge lamps for the adjacent lines are very weak by comparison.

A new Cenco Hyvac pump was used to exhaust the thermopiles. After the thermopiles were made vacuum tight there was practically no difference in readings between using the Cenco pump alone and when a mercury diffusion pump and a liquid air trap were used close to the thermopile. Pressures were of the order of a micron. Before the thermopile housings were made vacuum tight, residual pressures of the order of a mm of mercury caused the thermopiles to read approximately the same as though they were filled with air to atmospheric pressure.

Most of the radiant energy from a low pressure mercury discharge, such as a Sterilamp, is at 2537A. To extend the investigation to longer wave-lengths a low pressure neon lamp was made in the form of a pancake spiral. This lamp gave most of its energy in the 6500A region. A cozy-glow Nichrome wire heater element was selected as a source of infra-red and when operated at a temperature of about 900°K in conjunction with a Corning glass filter No. 254, 2 mm thick, gave its maximum energy near 2.8 mu. These three points, together with the data for the Standard carbon filament lamp furnish the points for the curve of Fig. 1.

RESULTS

Tables I and II record data obtained with evacuated and air-filled carbon-coated thermopiles on a number of lamps of different operating characteristics. In each case lower values were obtained when the thermopiles were evacuated. Energies are recorded as microwatts per cm² at one meter distance. Thermopile *A*, Table I, gave an average value to the ratio vac./air of 0.66 while thermopile *B*, Table II, indicated a ratio of 0.76. With a poor vacuum, i.e., air leakage through the gasket seal, thermopile *B* gave a ratio of about 0.96. With thermopiles *A* and *B* well evacuated the ratio for the neon lamp was always appreciably higher than for the ultraviolet lamps but still quite far from unity. Likewise the S1 sunlamp that radiated considerable infra-red gave a higher ratio value than for the S4 sunlamps. As measured with a monochromator or with a titanium photo-cell[3] the S1 and S4

TABLE I. *Energy measurements with thermopile "A."*

LAMP	CURRENT	EVACUATED	AIR-FILLED	RATIO VAC./AIR
S1 sunlamp	27.0 amp.	42.2	57.5	0.73
S4–1 sunlamp	.88	13.4	20.1	.67
S4–2 sunlamp	.88	15.2	22.1	.69
Low pressure Hg	.020	28.9	44.3	.65
Sterilamp—1	.040	10.0	16.0	.63
Sterilamp—2	.040	10.7	18.3	.59
Sterilamp—3	.040	13.6	23.4	.58
Neon lamp	.045	64.2	85.0	.76
				Av. 0.66
Standard lamp	.450	352	352	1.00
Nichrome heater unit (a)	3.0	291	258	1.13
Nichrome heater unit (b)	3.0	585	510	1.15

[1] W. E. Forsythe, *Measurement of Radiant Energy* (McGraw Hill, 1937); D. C. Stockbarger, "Filter radiometry," p. 98.

[2] F. Benford and R. F. Howe, Trans. I. E. S. **26**, 292 (1931).

[3] J. W. Marden and K. O. Smith, Trans. Elec. Chem. Soc. **62**, 237 (1932).

sunlamps have approximately the same amount of energy in the erythemal region, so this filter combination did not completely isolate the erythemal region. Since the thermopile was calibrated with the Standard lamp before each set of observations, the carbon filament lamp had a vac./air ratio of unity. The infra-red radiations from the Nichrome wire heater element produced a ratio greater than unity. Two values are recorded for this unit—one marked "a" in Tables I and IV refer to measurements made through both the quartz window on the thermopile and the Corning heat transmitting

TABLE II. *Energy measurements with thermopile "B."*

LAMP	EVACUATED	AIR	VAC./AIR	POOR VACUUM	AIR	VAC./AIR
S1 sunlamp	50.0	64.0	0.78	56.5	61.0	0.93
S4–1 sunlamp	16.1	22.1	.73	20.8	21.6	.96
S4–2 sunlamp	17.6	24.4	.72	22.9	23.2	.98
Low pressure Hg	32.2	46.0	.70	43.7	44.8	.96
Neon lamp	76.7	89.4	.86	85.5	88.5	.96
			Av. 0.76			Av. 0.96

filter, while in "b" only the quartz filter was in the optical beam and permitted the inclusion of a greater spectral band. A water cell screened off practically all radiations from this source.

When the large discrepancy was observed between the evacuated and air-filled thermopile readings, it was decided to try various gases such as carbon dioxide, sulfur dioxide, hydrogen and ammonia to see what effect gases of different molecular weights and different chemical properties might have on the surface of the thermopile elements. Table III shows that these various gases or vapors, with the exception of hydrogen gas, give approximately the same results as air. Hydrogen gas produced somewhat higher values for each lamp.

From an examination of the preceding data it was concluded that the difficulties in making reproducible measurements were due to the coating on the thermopile elements. To verify these suspicions thermopile "*A*" was sent to the manufacturer with a request that the carbon coating be removed and be replaced with platinum black. A new thermopile element was supplied that had been coated electrolytically with a deposit of platinum black. Data obtained with this element, thermopile "*C*," are recorded in Table IV. With this thermopile the readings are the same, whether evacuated or filled with air.

Figure 1 shows the relationship between the vac./air ratio for several lamps plotted against

TABLE III. *Readings with various gas fillings, thermopile "A."*

	S1 LAMP	S4–1 LAMP	HG LAMP	NEON LAMP
Carbon dioxide	60.8	20.2	48.8	85.5
Paradichlorbenzene	62.5	20.6	47.8	86.5
Acetone vapor	59.8	20.3	47.0	84.5
Ammonia	63.5	21.6	50.0	85.1
Sulphur dioxide	60.7	21.4	47.0	85.1
Hydrogen	70.5	24.8	53.4	89.0
Air	55.2	20.0	45.5	86.7

wave-length. The data were taken from Tables I and IV. Sterilamps give approximately monochromatic radiation at 2537A while the neon lamp gives out most of its energy near 6500A. Effective wave-lengths for the Standard carbon filament lamp and the Nichrome heater unit were assigned by consideration of the energy of a blackbody at 1700°K and 900°K, the latter passing through a Corning infra-red glass filter. The wave-length values are only approximate. The curve shows that there is a variation of almost 2 to 1 between the infra-red and ultraviolet region in the vac./air ratio as measured with thermopile "*A*." Thermopile "*C*" gives a ratio of unity over the spectral range .25μ to 3.0μ.

The method of determining an approximate wave-length for the effective 900°K radiation is shown in Fig. 2. The relative energy of the 900°K blackbody and the transmission of the Corning filter No. 254 are plotted against wave-length. The product of filter transmission and relative energy gives a curve from which the approximate average wave-length can be determined. The effective wave-length for the cozy glow heater element is near 2.8μ. An effective wave-length of 1.8μ was selected for the Standard carbon filament lamp burning at approximately 1700°K.

DISCUSSION

Apparently the gas filling the interstices between the carbon particles affects the relative amounts of ultraviolet and infra-red radiation measured by our thermopiles. One explanation is

that the radiations from the Standard lamp used in calibrating the thermopile, chiefly infra-red, are able to penetrate the sooted layer whether the thermopile is filled with gas or is evacuated. It is well known that carbon becomes increasingly more transparent at longer wave-lengths. The ultraviolet radiations, however, are absorbed by the outer layer of soot, the energy being transferred to the thermal junctions by conduction through the lower layers of carbon particles and by the gas molecules within the coating. When evacuated, the ultraviolet radiations are not completely transferred through the loose particles of the spongy carbon coating by conduction alone, and so only a part of the incident radiation is recorded. Thermopile "*A*" showing the greatest variation between air-filled and evacuated conditions seems to have a heavier coating of carbon than thermopile element, "*B*," but no measurement was made of its thickness. Because of the great thermal conductivity of hydrogen gas, one would expect this gas to be the most efficient in transferring heat from the outer layer of carbon to the metallic thermal junctions. On this basis the readings with the thermopile filled with hydrogen should be more nearly representative of the true values than those obtained with other gas fillings. The high thermal conductivity of hydrogen reduced the sensitivity of the thermopile to about 40 percent of its value when filled with air.

Since the ratio vac./air for the infra-red heater element was greater than unity for the carbon-coated thermopile junctions, radiations from the Standard carbon filament lamp as well as from all other lamps are partially transported from the outer coating to the thermopile elements by gas conduction in the coating layer. Hence this type of carbon coating on the thermopile elements is not satisfactory for accurate measurements. A much better absorber of radiant energy giving the thermopile a truly non-selective coating can be made with fine metallic powders such as platinum black or zinc black described by Pfund.[4] Data in Table IV confirm the above theory that a deposit of platinum black is a non-selective absorber of ultraviolet, visible and infra-red radiations and can be used for thermopile elements.

TABLE IV. *Energy measurements with thermopile "C."*

LAMP	CURRENT	EVACUATED	AIR-FILLED	RATIO VAC./AIR
S1 sunlamp	27.0 amp.	51.6	50.0	1.03
S4-1 sunlamp	.88	20.0	19.7	1.02
S4-2 sunlamp	.88	22.2	22.0	1.01
Low pressure Hg	.020	42.0	43.2	.97
Sterilamp—3	.040	24.8	25.1	.99
Neon Lamp	.045	89.3	87.3	1.02
Standard Lamp	.450	352	352	1.00
Nichrome heater unit (a)	3.0	390	395	.98
Nichrome heater unit (b)	3.0	580	585	.99

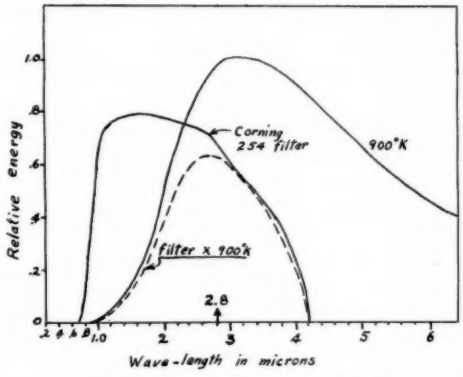

FIG. 2. The product of relative energy from a 900°K source and transmission of a Corning No. 254 filter gives an effective wave-length of about 2.8 microns for the Nichrome wire heater element.

Since the ratio of vacuum readings to air-filled readings is unity for the thermopile coated with platinum black, data obtained with this thermopile should be much more reliable than that obtained with a carbon-coated thermopile. With carbon-coated thermopiles the greater the difference in spectral energy distribution between a given lamp and the standard calibration lamp the greater will be the probable error when measurements are made with the thermopile when evacuated. Variations in energy distribution do not affect the thermopiles that are coated with a metallic coating such as zinc black or platinum black.

[4] A. H. Pfund, "Blackening of radiometers," reference 1, p. 210.

Note on the Evaluation of Photographic Speed from Sensitometric Data*

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(Received September 5, 1941)

A method of ascertaining the gradient speed of a photographic material without the use of special gradient measuring instruments is pointed out. The derived formula shows the relation which must exist between density and log exposure at the value of exposure which is the reciprocal of the speed.

IT has been demonstrated by Jones[1] that a reliable determination of the effective camera speed of a photographic negative material may be made by the following sensitometric procedure. The characteristic curve of the material is produced in the conventional manner (Fig. 1). Upon this curve is located a point, A, at which the slope of the tangent has a value that is a fractional part of a second gradient. This second gradient is the average gradient of that portion of the D-log E curve which is used in making the negative. In terms of the figure, the range employed is designated by A to C. The position of A is determined by the condition that the tangent of the angle a shall bear a certain relation to that of the angle b. The abscissa of A in log E units is the basis upon which the speed rating is made.

It has recently been proposed as an American Standard[2] that the reciprocal of the exposure at A, in meter-candle-seconds, shall be the speed. Specific conditions as set up are that the log E distance from A to C shall be 1.5 and that the tangent of angle a shall be 0.3 times that of angle b.

It is obvious that the direct method of evaluating photographic speed according to this criterion involves the measurement of two gradient values—a procedure that is perhaps not commonly practiced in laboratories doing sensitometric work. The author[3] at one time suggested designs for various instruments with which the desired data could be readily obtained. In that paper it was also pointed out how the whole complicated operation of analyzing a sensitometric strip and computing relative gradients can be performed automatically with a photoelectric densitometer.

The author realizes that the instruments suggested are special tools requiring some expenditure of time and money to produce. For this reason the comparatively simple suggestion of an accurate and not laborious mathematical method of analyzing conventional sensitometric data may prove useful.

The condition for the establishment of the speed value stated algebraically is

$$G_{\text{limit}} = 0.3\, G_{\text{average}}, \qquad (1)$$

in which G_{limit} is theoretically the first derivative

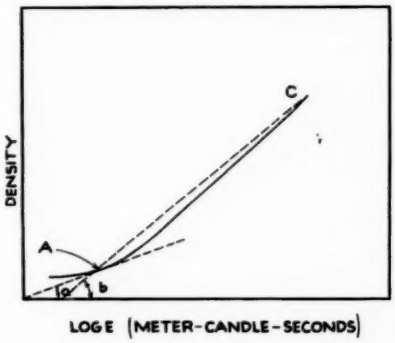

FIG. 1. Method of determination of gradient speed point.

* Communication No. 818 from the Kodak Research Laboratories.

[1] L. A. Jones, "The evaluation of negative film speeds in terms of print quality," J. Frank. Inst. **227**, 297 (1939); **227**, 497 (1939).

[2] American Standards Association, Committee on Standardization in the Field of Photography, Subcommittee on Sensitivity to Radiant Energy. "Proposed American Standard: Specifications for determining photographic speeds of roll films, film packs, and miniature camera films," J. Opt. Soc. Am. **31**, 87 (1941).

[3] C. Tuttle, "Methods and instruments for the determination of photographic speeds by measurement of relative characteristic gradients," J. Opt. Soc. Am. **29**, 267 (1939).

of the density-log E function and $G_{average}$ is by the specified condition the density difference between A and C in Fig. 1 divided by 1.5.

Actually, with typical negative emulsions, it is not necessary to find G_{limit}, as the theoretical $dD/d \log E$ for this first derivative value is very closely approximated by the ratio of a finite density increment to a finite log E increment, even if the log E increment is as large as 0.3. Thus, in place of G_{limit} we can write $(D_2-D_1/0.3)$, where D_1 and D_2 are densities on the toe of the curve produced by two log exposures separated by 0.3.

The gradient value so found would apply to a point on the characteristic curve midway between D_1 and D_2. The density at this midpoint would be the average of D_1 and D_2—again an approximation, but a very close one, for the photographic characteristic curve.

For the value of $G_{average}$, may be written the equation

$$D_3-\tfrac{1}{2}(D_1+D_2)\div 1.5,$$

in which D_3 is a density produced by a log exposure 1.5 greater than that which produced the density midway between D_1 and D_2.

With these substitutions, Eq. (1) may be rewritten:

$$\frac{D_2-D_1}{0.3}=0.3\left[D_3-\frac{D_1+D_2}{2}\right]\Big/1.5. \qquad (2)$$

This simplifies to

$$17.18D_2-16.18D_1=D_3. \qquad (3)$$

One further approximation is allowable. Since D_1 and D_2 are always on the toe of the curve, and therefore both numerically small, the decimal part (0.18) of the coefficient on D_1 and D_2 may be dropped, so that an algebraic relation presenting no arithmetic difficulties is the result:

$$17D_2-16D_1=D_3. \qquad (4)$$

By the use of this equation sensitometric data, as usually obtained for the plotting of characteristic curves, may be used conveniently to establish the gradient speed value precisely and without recourse to any special gradient-measuring device. Take as an example the typical data for a negative material sensitometric strip shown in Table I. Column 1 shows the step

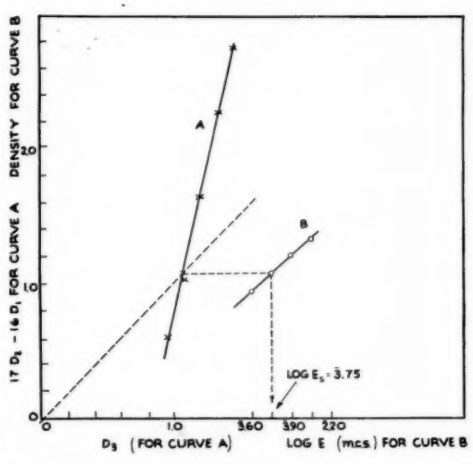

FIG. 2. Gradient speed determination by plotting data derived from density reading.

numbers, column 2, the actual log E values in meter-candle-seconds, and column 3, the step densities. Now, the values in column 3 obviously may be used to form a series of D_1 and D_2 values. For example, let step 21 be the first D_1 value; the corresponding D_2 value will be two 0.15 steps farther along in the table, which is step 19. Now, the steps 19 and 21 really determine the gradient at step 20; hence, the first D_1 and D_2 values are entered opposite step 20. Columns 6, 7, and 8 are self-explanatory. The last column, D_3, is taken from column 3, starting at the 11th step, thus advancing D_3 to a position ten 0.15 log E steps from the points midway between D_1 and D_2.

A cursory examination of the last two columns reveals that they are approximately equal at step 19, and the log E value from which the speed is to be determined should be about $\bar{3}.75$. To accept this as the true value, or even to interpolate in the table to ascertain the value, would be placing more than necessary reliance upon the densitometric accuracy for two D_1 and D_2 values. It is better to follow the procedure shown in Fig. 2 and to plot the points in column 8 against those in column 9. It will be noted that these points rather definitely establish the position and slope of a straight line (A). A dotted line at 45 degrees is drawn to establish the location of the point at which $17D_2-16D_1$ is equal to D_3. In this particular case, the value happens to be exactly 1.08—identical with the D_3 value

for step 19. The log exposure for the gradient speed determination is $\bar{3}.75$, as previously estimated. In general to find the value of log E which applies to a particular value of D_3, it is desirable to construct a short section of a second curve by plotting a few values of column 2 against values in column 9. This has been done in B of Fig. 2. This short segment of a curve may always be considered a straight line. By interpolation on this curve the log E value may be readily found.

To check the results of the formula, the gradient speed value of this particular sample was compared with that given by other methods of gradient speed determination. The numerical results of all the most precise methods proved to be in excellent agreement with that found here.

The particular set of data given in Table I was the product of very careful sensitometric and densitometric technique. Moreover, the data apply to a normal kind of negative emulsion curve with a typical toe shape and a long straight-line portion. Such perfect correlation of speed values and such easily drawn curves as those of Fig. 2 are hardly to be expected in routine work. If the sensitometric and densitometric work is less refined or if emulsions of less conventional characteristic shapes are encountered, disagreement will probably result.

To test the general usefulness of the formula, samples of negative emulsions covering the gamut of present commercial emulsion curve shapes were obtained. Sensitometric data were then collected for these samples by the group of these Laboratories that deals with routine testing. The data were then subjected to the formula for the determination of the log E_s values. They were also treated by an older, thoroughly tested method involving the use of a transparent scale known and distributed as the "fractional gradient speed meter." This meter measures directly on a characteristic curve plotted from the data and gives, in the hands of experienced users, entirely consistent results. For a description of the method see the paper previously cited.[3]

TABLE I. *Sensitometric data and computed values.*

STEP	LOG E	D	D_1	D_2	$16D_1$	$17D_2$	$17D_2-16D_1$	D_3
21	$\bar{3}.45$	0.10						0.83
20	3.60	0.11	0.10	0.13	1.60	2.21	0.61	0.95
19	3.75	0.13	0.11	0.165	1.76	2.80	1.04	1.08
18	$\bar{3}.90$	0.165	0.13	0.22	2.08	3.74	1.66	1.21
17	$\bar{2}.05$	0.22	0.165	0.29	2.64	4.93	2.29	1.34
16	2.20	0.29	0.22	0.37	3.52	6.29	2.77	1.46
15	2.35	0.37						
14	2.50	0.47						
13	2.65	0.59						
12	2.80	0.70						
11	$\bar{2}.95$	0.83						
10	$\bar{1}.10$	0.95						
9	1.25	1.08						
8	1.40	1.21						
7	1.55	1.34						
6	1.70	1.46						

A comparison of log E_s values found by the formula and by the gradient scale meter is given in Table II, along with a general description of the character of each curve and the value of gamma for each sample. The greatest error is 0.03 in log E which is about 7 percent in speed—approximately a quarter-camera-lens stop.

TABLE II. Log E_s *determined by different methods.*

SAMPLE	DESCRIPTION	γ	LOG E_s (FORMULA)	LOG E_s (SCALE)
A	Long toe	0.45	$\bar{3}.41$	$\bar{3}.38$
B	Same material as *A*	1.15	$\bar{3}.25$	$\bar{3}.28$
C	Normal shape	0.8	$\bar{3}.75$	$\bar{3}.75$
D	Normal shape	0.5	$\bar{3}.43$	$\bar{3}.43$
E	Short toe	1.3	$\bar{3}.23$	$\bar{3}.26$

The sole advantage claimed for the use of the formula has been that the speed value may be determined from conventional sensitometric data without resort to any auxiliary equipment. It might be mentioned, however, that the arithmetic involved would be materially shortened if the densitometer were calibrated to read $16D$ and $17D$ directly. If it is desired to elaborate further on an instrument for use with the formula it would be possible to incorporate a simple computer to subtract $16D_1$ from $17D_2$.

The accuracy and reliability of any method of speed evaluation are influenced by the following uncertainties:

1. The photometric inaccuracies of densitometry.
2. The unevenness of emulsion coating, particularly in that portion of the sensitometric strip which bears the low-density image.
3. The inequalities of sensitometric strip processing.
4. The uncertainty that a representative sample is taken from the emulsion batch.
5. The errors in sensitometric exposure caused by fluctuating lamp intensity.

All of these uncertainties are decreased by taking multiple samples and by averaging the results.

In cases where the importance of the results justifies multiple sampling, there are some interesting possibilities in the application of the formula to obtaining the gradient speed value directly. This formula may be written:

$$D_3+16D_1=17D_2.$$

Suppose that a sensitometer such as the Eastman IIb instrument is altered slightly so that one-half of the strip, divided lengthwise, receives the normal exposure, while the other half is given twice as much exposure. Each step of the processed strip will then consist of two areas, the lighter of which represents D_1, while the darker represents D_2. If sixteen such strips are stacked in a pile with all steps in register, the densities will be additive, so that densitometer readings of the stack would give directly $16D_1$ and $16D_2$.* Now, if a seventeenth sample is exposed on the sensitometer to give one side a twice-normal exposure and the other side a forty-five times normal exposure, the two halves of the strip will represent D_2 and D_3, respectively. If this last strip is superimposed upon the stack of sixteen, the conditions required by the formula will be fulfilled. The reading of a stack of densities, as was pointed out by the author,[3] will very much diminish the probable error of the densitometry in the toe region and at the same time will minimize the other sources of error.

It is of interest to mention that with such a stack of samples it is possible to assign the speed value approximately without the aid of even a densitometer. If the stack of strips is illuminated by a strong light, the lowest steps of the D_2 group are definitely more transparent than the adjacent area of the other group, but the gradient of this side of the strip is materially greater than the gradient of the other side, so that within a few steps it becomes definitely the darker side. There probably would never be a case in which the D_2 side, as judged by the unaided eye, would not change from the darker to the lighter in the increment of one step, so that it is safe to say that speed can be *estimated* to within 0.15 in log E, by this method.

The more accurate method of comparison is, of course, to read a few steps of each half with a densitometer, and then, after plotting each set of densities against log E to determine the speed value from the point of intersection of the two curves. Some modification of the conventional densitometer to enable it to read higher densities than usual would be found necessary.

* The validity of the addition of densities by superposition has been questioned on the grounds that interreflections between layers would affect the numerical result. The procedure is justified in this case because one is interested only in the comparison of an equal number of layers at an equal density value. The surfaces concerned have nearly equal reflectances so the error if any is cancelled.